



# On the thermodynamics of hairy black holes



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## ABSTRACT

We investigate the thermodynamics of a general class of exact 4-dimensional asymptotically Anti-de Sitter hairy black hole solutions and show that, for a fixed temperature, there are small and large hairy black holes similar to the Schwarzschild–AdS black hole. The large black holes have positive specific heat and so they can be in equilibrium with a thermal bath of radiation at the Hawking temperature. The relevant thermodynamic quantities are computed by using the Hamiltonian formalism and counterterm method. We explicitly show that there are first order phase transitions similar to the Hawking–Page phase transition.

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## 1. Introduction

Asymptotically Anti-de Sitter (AdS) black holes play an important role in understanding the dynamics and thermodynamics of holographic dual field theories via AdS/CFT duality [1]. In particular, these black holes are dual to thermal states of the ‘boundary’ field theory. First order phase transitions in the bulk can be related to confinement/deconfinement-like phase transitions in the dual field theory [2]. Since scalar fields appear as moduli in string theory, it is important to understand the generic thermodynamic properties of hairy black holes.

Motivated by these considerations, in this paper we study in detail the thermodynamics of a general class of exact 4-dimensional neutral hairy black holes [3–5] (examples in other dimensions or for different horizon topologies can be found in [6–13]). The scalar potential is characterized by two parameters and the black hole solution has one integration constant that is related to its mass. For some particular values of the parameters in the potential, the solutions can be embedded in supergravity [5,6]. The scalar field potential contains as special cases all the uncharged exact static solutions so far discussed in the literature [14–16] (for details, see [17]). These static configurations have been extended to dynamical black hole solutions [18,19].

There are some subtleties in defining the mass of a hairy black hole [20–24]. In [24], a concrete method of computing the mass of

an asymptotically AdS hairy black hole was proposed. This method is very useful from a practical point of view because it is using just the expansions of the metric functions at the boundary. More importantly, it can be used for hairy black holes that preserve or not the conformal symmetry (the AdS isometries) of the boundary. We are going to use this method, which is based on the Hamiltonian formalism [25], to compute the mass of the black hole solutions.

However, based on the physics of AdS/CFT duality, a different method was developed, the so-called ‘holographic renormalization [26] (see, also, [27–31]<sup>1</sup>) – for boundary mixed conditions for the scalar field, this method was further developed in [36,37]. The main idea behind this method is that, due to the holography, the infrared (IR) divergences that appear in the gravity side are equivalent with the ultraviolet divergences of the dual field theory. Then, to cure these divergences, one needs to add counterterms that are local and depend on the intrinsic geometry of the boundary. In this way, one can use the quasilocal formalism of Brown and York [38] supplemented with these counterterms to compute the regularized Euclidean action and the ‘boundary’ stress tensor. The energy is the charge associated with the Killing vector  $\partial_t$  and it can be obtained from the boundary stress tensor.

Armed with these results, one can investigate the thermodynamics and phase diagram of the hairy black hole solutions. In particular, we show that there are first order phase transitions that lead to a discontinuity in the entropy. Similar results were

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<sup>1</sup> A similar method for asymptotically flat spacetimes was developed in [32,33] and some concrete applications were presented in [34,35].

obtained for a general class of black holes solutions in a theory with a conformal invariant scalar field Lagrangian [39].

The rest of the paper is organized as follows. In Section 2, we review the exact hairy black hole solutions and briefly present some of their properties. Section 3 is dedicated to the computations of the thermodynamic quantities. To gain some intuition, we present a detailed computation of Schwarzschild–AdS (SAdS) black hole in the coordinates the hairy solution was written. Then, to regularize the Euclidean action (and the boundary stress tensor), we propose a counterterm that depends on the scalar field and it is intrinsic to the boundary. The mass is computed with the counterterm formalism and, also, by the method of [24]. In Section 4, we use the results in the previous section to investigate the existence of the phase transitions. Then, Section 5 concludes with a summary of results.

## 2. Black hole solution

We are interested in asymptotically AdS hairy black hole solutions with a spherical horizon [4,5]. The action is

$$I[g_{\mu\nu}, \phi] = \int_{\mathcal{M}} d^4x \sqrt{-g} \left[ \frac{R}{2\kappa} - \frac{(\partial\phi)^2}{2} - V(\phi) \right] + \frac{1}{\kappa} \int_{\partial\mathcal{M}} d^3x K \sqrt{-h} \quad (1)$$

where  $V(\phi)$  is the scalar potential,  $\kappa = 8\pi G_N$ , and the last term is the Gibbons–Hawking boundary term. Here,  $h_{ab}$  is the boundary metric and  $K$  is the trace of the extrinsic curvature. The metric ansatz is

$$ds^2 = \Omega(x) \left[ -f(x) dt^2 + \frac{\eta^2 dx^2}{f(x)} + d\theta^2 + \sin^2\theta d\phi^2 \right] \quad (2)$$

We consider the following scalar potential, which for some particular values of the parameters it becomes the one of a truncation of  $\omega$ -deformed gauged  $N = 8$  supergravity [5,40,41]:

$$V(\phi) = \frac{\Lambda(\nu^2 - 4)}{6\kappa\nu^2} \left[ \frac{\nu - 1}{\nu + 2} e^{-\phi l_\nu(\nu+1)} + \frac{\nu + 1}{\nu - 2} e^{\phi l_\nu(\nu-1)} + 4 \frac{\nu^2 - 1}{\nu^2 - 4} e^{-\phi l_\nu} \right] + \frac{\alpha}{\kappa\nu^2} \left[ \frac{\nu - 1}{\nu + 2} \sinh \phi l_\nu(\nu + 1) - \frac{\nu + 1}{\nu - 2} \sinh \phi l_\nu(\nu - 1) + 4 \frac{\nu^2 - 1}{\nu^2 - 4} \sinh \phi l_\nu \right] \quad (3)$$

The equations of motion can be integrated for the conformal factor [4,7,9,11]:

$$\Omega(x) = \frac{\nu^2 x^{\nu-1}}{\eta^2 (x^\nu - 1)^2} \quad (4)$$

where  $\alpha$  and  $\nu$  are two parameters that characterize the hairy solution. With this choice of the conformal factor, it is straightforward to obtain the expressions for the scalar field

$$\phi(x) = l_\nu^{-1} \ln x \quad (5)$$

and metric function

$$f(x) = \frac{1}{l^2} + \alpha \left[ \frac{1}{\nu^2 - 4} - \frac{x^2}{\nu^2} \left( 1 + \frac{x^{-\nu}}{\nu - 2} - \frac{x^\nu}{\nu + 2} \right) \right] + \frac{x}{\Omega(x)} \quad (6)$$

where  $\eta$  is the only integration constant and  $l_\nu^{-1} = \sqrt{(\nu^2 - 1)/2\kappa}$ .

The potential and the solution are invariant under the transformation  $\nu \rightarrow -\nu$ . For  $x = 1$ , which corresponds to the boundary, we can show that the theory has a standard AdS vacuum

$V(\phi = 0) = \frac{\Lambda}{\kappa}$ . In the limit  $\nu = 1$ , one gets  $l_\nu \rightarrow \infty$  and  $\phi \rightarrow 0$  so that the SAdS black hole is smoothly obtained.

The mass of the scalar field can be easily computed from the expansion of the potential and we obtain  $m^2 = -2/l^2$ , which is the ‘conformal’ mass. It is also important to point out that there are two distinct branches, one that corresponds to  $x \in [0, 1]$  and the other one to  $x \in [1, \infty]$  – the boundary is at  $x = 1$  and the curvature singularities are at  $x = 0$  for the first branch and  $x \rightarrow \infty$  for the second one (these are the locations where the scalar field is also blowing up).

## 3. Thermodynamics

In this section we use the quasilocal formalism supplemented with counterterms to compute the Euclidean action and hairy black hole’s energy. For completeness, we also compute the mass with the method of [24] that is based on the Hamiltonian formalism [25].

As a warm-up example, let us start with SAdS black hole in the coordinates (2) that can be obtained when the hair parameter is  $\nu = 1$ . The metric (2) becomes in this case

$$\Omega(x) = \frac{1}{\eta^2 (x - 1)^2}, \quad f(x) = \frac{1}{l^2} + \frac{1}{3} \alpha (x - 1)^3 + \eta^2 x (x - 1)^2 \quad (7)$$

To obtain the SAdS black hole in the canonical form, one has to change the coordinates as

$$x = 1 \pm \frac{1}{\eta r} \quad (8)$$

The reason is that, as we have already discussed, there are two branches that correspond to  $x \in [0, 1]$  and  $x \in [1, \infty]$ . Since there are some subtleties for computing the action for  $x \in [1, \infty]$  branch (for example, the extrinsic curvature is changing the sign due to a change of the normal to the foliation  $x = \text{constant}$ ), in what follows we explicitly work with the branch  $x \in [0, 1]$ . In this case, using the change of coordinates (8) we obtain the SAdS black hole in canonical coordinates:

$$\Omega(x) f(x) = F(r) = 1 - \frac{\mu}{r} + \frac{r^2}{l^2}, \quad \mu = \frac{\alpha + 3\eta^2}{3\eta^3} \quad (9)$$

It is well known that the action has divergences even at the tree level due to the integration on an infinite volume. To regularize the action, we use the counterterms [27]:

$$I[g_{\mu\nu}] = I_{\text{bulk}} + I_{\text{GH}} - \frac{1}{\kappa} \int_{\partial\mathcal{M}} d^3x \sqrt{-h} \left( \frac{2}{l} + \frac{\mathcal{R}l}{2} \right) \quad (10)$$

where  $\mathcal{R}$  is the Ricci scalar of the boundary metric  $h_{ab}$ .

Let us first compute the bulk action – in this case, since the scalar field vanishes the potential becomes the cosmological constant:  $V = \frac{\Lambda}{\kappa} = -\frac{3}{l^2\kappa}$ . We use the trace of the Einstein tensor and following combinations of the equations of motion

$$E_t^t - E_\phi^\phi = 0 \Rightarrow 0 = f'' + \frac{\Omega' f'}{\Omega} + 2\eta^2 \quad (11)$$

$$E_t^t + E_\phi^\phi = 0 \Rightarrow 2\kappa V(\phi) = -\frac{(f\Omega'' + f'\Omega')}{\Omega^2 \eta^2} + \frac{2}{\Omega}$$

to obtain

$$I_{\text{bulk}}^E = \frac{4\pi\beta}{\eta^3 \kappa l^2} \left[ -\frac{1}{(x_b - 1)^3} + \frac{1}{(x_h - 1)^3} \right] = \frac{4\pi\beta}{\kappa l^2} (r_b^3 - r_h^3) \quad (12)$$

Here,  $x_b$  and  $x_h$  are the boundary and horizon locations, and  $\beta$  is the periodicity of the Euclidean time that is related to the temperature by  $\beta = T^{-1}$ .

The Gibbons–Hawking surface term can be computed if we choose a foliation  $x = \text{constant}$  with the induced metric  $ds^2 = h_{ab}dx^a dx^b = \Omega(x)[-f(x)dt^2 + d\theta^2 + \sin^2\theta d\phi^2]$ . The normal to the surface  $x = \text{constant}$  and extrinsic curvature are

$$n_a = \frac{\delta_a^x}{\sqrt{g^{xx}}}, \quad K_{ab} = \frac{\sqrt{g^{xx}}}{2} \partial_x h_{ab} \quad (13)$$

and the contribution of the Gibbons–Hawking term to the action is

$$\begin{aligned} I_{GH}^E &= -\frac{2\pi\beta}{\kappa} \left[ -\frac{6}{l^2\eta^3(x-1)^3} - \frac{4}{\eta(x-1)} - \left( \frac{\alpha + 3\eta^2}{\eta^3} \right) \right] \Big|_{x_b} \\ &= -\frac{2\pi\beta}{\kappa} \left( \frac{6r_b^3}{l^2} + 4r_b - 3\mu \right) \end{aligned} \quad (14)$$

The last contribution is given by the gravitational counterterm, which is an intrinsic surface term that depends only on the geometry of the boundary

$$\begin{aligned} I_{ct}^E &= \frac{2\pi\beta}{\kappa} \left[ \frac{4}{l^2\eta^3(x_b-1)^3} + \frac{4}{\eta(x_b-1)} - 2\mu \right] \\ &= \frac{2\pi\beta}{\kappa} \left( \frac{4r_b^3}{l^2} + 4r_b - 2\mu \right) \end{aligned} \quad (15)$$

We can explicitly see that the divergences proportional with  $r_b \rightarrow \infty$  and  $(r_b)^3 \rightarrow \infty$  cancel out and so the regularized action is

$$\begin{aligned} I^E &= I_{bulk}^E + I_{GH}^E + I_{ct}^E = \frac{4\pi\beta}{\kappa l^2} \left[ \frac{1}{\eta^3(x_h-1)^3} + \frac{\mu l^2}{2} \right] \\ &= \frac{4\pi\beta}{\kappa l^2} \left( -r_h^3 + \frac{\mu l^2}{2} \right) \end{aligned} \quad (16)$$

The computations for the general hairy black hole (2), (4), (6) are more involved but similar with the ones above and we do not present all the details here. In this case, the action should be supplemented with a counterterm that depends also on the scalar field [26,29,36,37].<sup>2</sup> We work with a counterterm that is intrinsic to the boundary geometry (it does not depend on the normal to the boundary or the normal derivatives of the scalar field) [36, 37]:

$$\begin{aligned} I_\phi^E &= \int_{\partial\mathcal{M}} d^3x^E \sqrt{h^E} \left( \frac{\phi^2}{2l} - \frac{l_v}{6l} \phi^3 \right) \\ &= \frac{4\pi\beta}{\kappa} \left[ -\frac{v^2-1}{4l^2\eta^3(x_b-1)} + \frac{v^2-1}{3l^2\eta^3} \right] \end{aligned} \quad (17)$$

The sum of the other terms in the action is

$$\begin{aligned} I_{bulk}^E + I_{surf}^E + I_{ct}^E &= -\frac{1}{T} \left( \frac{AT}{4G} \right) + \frac{4\pi\beta}{\kappa} \left[ \frac{v^2-1}{4l^2\eta^3(x_b-1)} \right. \\ &\quad \left. + \frac{12\eta^2 l^2 + 4\alpha l^2 - 4v^2 + 4}{12l^2\eta^3} \right] \end{aligned} \quad (18)$$

where  $A = 4\pi\Omega(x_h)$  is the area of the horizon. It is worth mentioning that the gravitational counterterm [27] is not sufficient to

cancel the divergence in the action (there is still a term proportional to  $(x_b-1)^{-1}$ ) but when we add the counterterm (17) we obtain a finite action:

$$I^E = \beta \left( -\frac{AT}{4G} + \frac{4\pi}{\kappa} \frac{3\eta^2 + \alpha}{3\eta^3} \right) \quad (19)$$

In the classical limit, the action is related to the thermodynamic potential (the free energy  $F$  in our case), which is  $F = I^E/\beta = M - TS$ . Using the well-known thermodynamic relations or by comparing the two formulas, one can extract the mass of the hairy black hole:

$$M = \frac{1}{2G} \left( \frac{\alpha + 3\eta^2}{3\eta^3} \right) \quad (20)$$

Since we have constructed the regularized action, we can use the quasilocal formalism of Brown and York [38] to construct the boundary stress tensor, which is the variation of the action with respect to the induced metric:

$$\begin{aligned} \tau_{ab} &= -\frac{1}{\kappa} \left( K_{ab} - h_{ab}K + \frac{2}{l} h_{ab} - lE_{ab} \right) \\ &\quad - \frac{h_{ab}}{l} \left( \frac{\phi^2}{2} - \frac{l_v}{6l} \phi^3 \right) \end{aligned} \quad (21)$$

The boundary metric can be locally written in ADM-like form:

$$h_{ab}dx^a dx^b = -N^2 dt^2 + \sigma_{ij}(dy^i + N^i dt)(dy^j + N^j dt) \quad (22)$$

where  $N$  and  $N^i$  are the lapse function and the shift vector respectively and  $y^i$  are the intrinsic coordinates on a (closed) hypersurface  $\Sigma$ . The boundary geometry has an isometry generated by the Killing vector  $\epsilon^a = (\partial_t)^a$  for which the conserved charge is the mass:

$$\begin{aligned} M &= Q_{\frac{\partial}{\partial t}} = \oint_{\Sigma} d^2y \sqrt{\sigma} n^a \tau_{ab} \epsilon^b = \sigma \Omega^{1/2} f^{-1/2} \tau_{tt} \Big|_{x_b} \\ &= \frac{4\pi}{\kappa} \left[ \frac{\alpha + 3\eta^2}{3\eta^3} + O(x-1) \right] \end{aligned} \quad (23)$$

where  $n^a = (\partial_t)^a / \sqrt{-g_{tt}}$  is the normal unit vector to the surface  $t = \text{constant}$ .

We can also obtain the mass by using the method of [24]. With the change of coordinates

$$x = 1 - \frac{1}{\eta r} + \frac{(v^2-1)}{24\eta^3 r^3} \left[ 1 + \frac{1}{\eta r} - \frac{9(v^2-9)}{80\eta^2 r^2} \right] + O(r^{-6}) \quad (24)$$

we can read off the mass from the subleading term of  $g_{tt}$ :

$$-g_{tt} = f(x)\Omega(x) = \frac{r^2}{l^2} + 1 + \frac{\alpha + 3\eta^2}{3\eta^3 r} + O(r^{-3}) \quad (25)$$

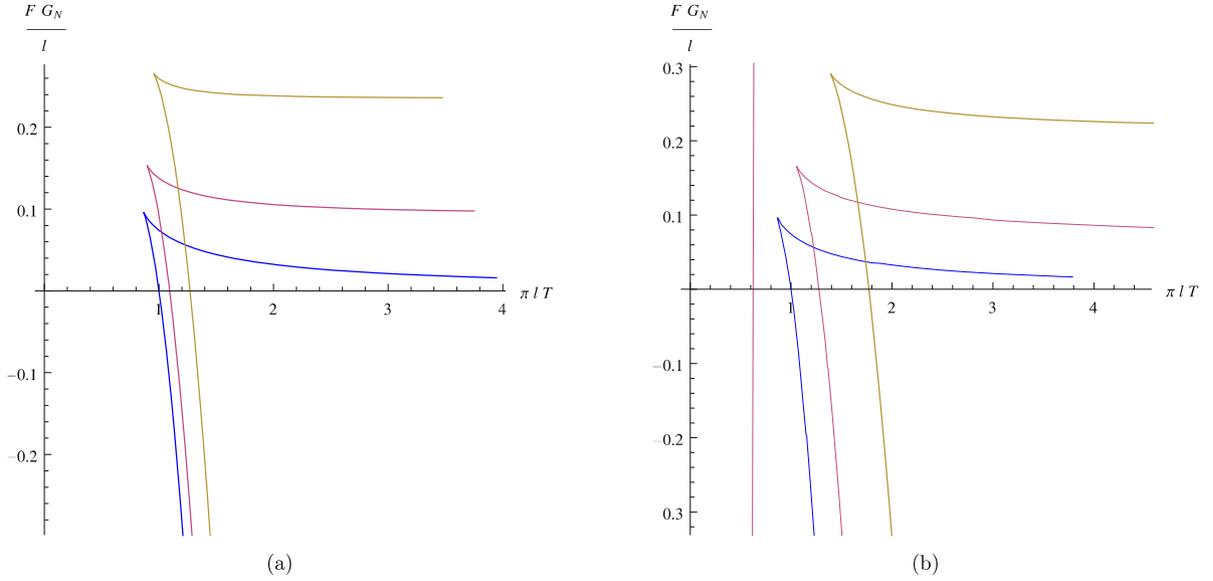
The reason is that the asymptotic expansion of the scalar field becomes in these coordinates

$$\phi(x) = l_v^{-1} \ln x = -\frac{1}{l_v \eta r} - \frac{1}{2l_v \eta^2 r^2} + \frac{v^2-9}{24\eta^3 r^3} + O(r^{-4}) \quad (26)$$

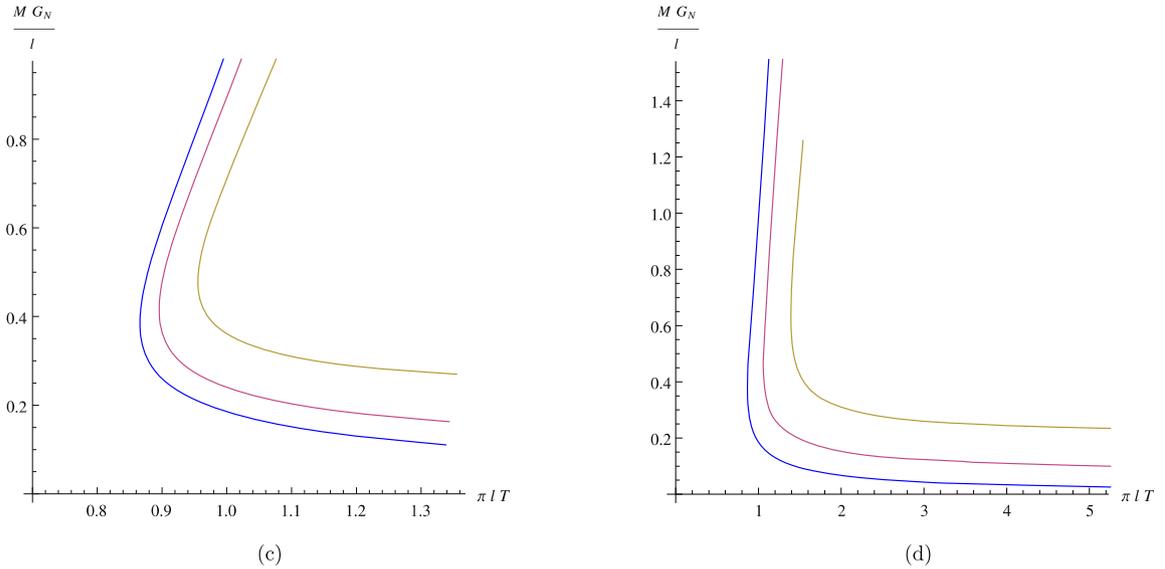
and we obtain that the coefficient of the leading term is  $-(l_v \eta)^{-1}$  and the subleading term is  $-(2l_v \eta^2)^{-1}$ . Both modes are normalizable and correspond to a solution for which the conformal symmetry of the boundary is preserved. Therefore, we obtain

$$M = \frac{1}{2G} \left( \frac{\alpha + 3\eta^2}{3\eta^3} \right) \quad (27)$$

<sup>2</sup> A more detailed analysis including concrete counterterms for (non-)logarithmic branch and a comparison with the Hamiltonian formalism is going to be presented in [37].



**Fig. 1.** (a) Dimensionless free energy versus dimensionless temperature, for different values of  $\nu$  and fixed  $\alpha = -10l^{-2}$  and positive scalar field. The plots are for  $\nu = 1$ ,  $\nu = 1.9$  and  $\nu = 3$  (from down up). The free energy of Schwarzschild AdS ( $\nu = 1$ ) goes to zero when  $T$  goes to infinity. The free energy of the hairy black holes goes to a constant at infinite temperature. (b) Dimensionless free energy versus dimensionless temperature, for different values of  $\nu$  and fixed  $\alpha = 10l^{-2}$  and negative scalar field. The plots are for  $\nu = 1$ ,  $\nu = 1.9$  and  $\nu = 3$  (from down up).



**Fig. 2.** (c) Dimensionless mass versus dimensionless temperature, for different values of  $\nu$  and fixed  $\alpha = 10l^{-2}$  and negative scalar field. The plots are for  $\nu = 1$ ,  $\nu = 1.9$  and  $\nu = 3$  (from left to right). Here it is possible to see the existence of small and large black hairy black holes in exact resemblance with Schwarzschild AdS. (d) Dimensionless mass versus dimensionless temperature, for different values of  $\nu$  and fixed  $\alpha = -10l^{-2}$  and positive scalar field. The plots are for  $\nu = 1$ ,  $\nu = 1.9$  and  $\nu = 3$  (from left to right). Here it is possible to see the existence of small and large black hairy black holes in exact resemblance with Schwarzschild AdS.

and this result matches the mass computed above with the quasilocal formalism. Using the following expressions for the temperature and entropy

$$T = \left. \frac{f'(x)}{4\pi\eta} \right|_{x=x_h} = \frac{1}{4\pi\eta\Omega(x_h)} \left[ \frac{\alpha}{\eta^2} + 2 + \nu \frac{x_h^\nu + 1}{x_h^\nu - 1} \right],$$

$$S = \frac{A}{4G} = \frac{4\pi\Omega(x_h)}{4G} \quad (28)$$

one can easily check that the first law  $dM = TdS$  is satisfied.

As we have already mentioned, the solutions fall into two distinct classes. For the family with a positive scalar field the mass is

$$M = -\frac{1}{2G} \left( \frac{\alpha + 3\eta^2}{3\eta^3} \right) \quad (29)$$

and temperature

$$T = -\frac{1}{4\pi\eta\Omega(x_h)} \left[ \frac{\alpha}{\eta^2} + 2 + \nu \frac{x_h^\nu + 1}{x_h^\nu - 1} \right] \quad (30)$$

with the entropy given by the area law.

#### 4. Phase transitions

The AdS spacetime can be thought to have a potential wall as one approaches the asymptotic infinity and it behaves as an infinite box (more rigorously, it has a conformal boundary). Since it is

not a globally hyperbolic spacetime the information can leak out or get in through the boundary and so, to obtain a well-defined problem, one has to impose boundary conditions.

The scalar field satisfies different boundary conditions depending on whether it is positive or negative, which corresponds to the two families of solutions mentioned before. A classical field theory is completely defined when the boundary conditions are prescribed. For any boundary conditions on the scalar field, the non-trivial vacuum configuration given by SAdS black hole solution should be included as an allowed state of the theory. Hence, in the canonical ensemble, its free energy can be compared with the one of the hairy black hole at a given temperature. Fig. 1(a) shows that, for the family with a positive scalar field, SAdS is always more favorable than the hairy configuration. Fig. 1(b) shows the same phenomena for the family with a negative scalar field. We found that generic values of  $\alpha$  do not change the qualitative behavior of the phase diagrams.

As in the SAdS case, there are two branches consisting of large and smaller black holes. Figs. 2(a) and 2(b) show the mass versus the temperature for the families with a positive scalar field and a negative one, respectively. These plots provide information about the specific heat

$$C = \frac{\partial M}{\partial T} \quad (31)$$

that is interpreted as the slope.

The entire branch of smaller black holes (for both families) is unstable thermodynamically and has a positive free energy, while the large black holes branch are stable thermodynamically and the free energy goes negative for all  $T > T_c$ . Unlike the planar black holes for which do not exist first order phase transitions with respect to AdS, the free energy is changing the sign. This is an indication that, for hairy black hole solutions with spherical horizon geometry, there are first order phase transitions with respect to thermal AdS — the large black hole solutions that have a negative free energy with respect to AdS are clearly the preferred ones.

## 5. Conclusions

From the point of view of AdS/CFT duality, the study of the thermodynamics of asymptotically AdS black holes is relevant to understanding the phase diagram of some holographic dual field theories. We have investigated the thermodynamics of a general class of hairy black holes with boundary conditions for a scalar field with the conformal mass  $m = -2/l^2$ , which preserve the AdS isometries. It is worth remarking the close similarity that we have observed with the familiar structure of SAdS black hole. The large hairy black holes are thermodynamically stable, and the smaller ones have a negative specific heat.

We have computed the Euclidean action (and so the thermodynamic potential) by using the quasilocal formalism supplemented with counterterms. Using these results, we have shown that there exist first order phase transitions between the thermal AdS and hairy black hole. On the other hand, by comparing the free energy of the hairy black hole with the one for the SAdS solution, it seems that the SAdS black hole is always preferred.

An interesting future direction is to check the existence of gravitational solitons as in [42] and the implications for the phase diagram. It will be also interesting to study the phase diagram of the family of exact charged hairy black holes presented in [4]. In this case, one can study both, the canonical and grand-canonical ensemble, respectively. In the canonical ensemble the charge, which is an extensive variable, should be kept fixed. Since AdS spacetime with a fixed charge is not a solution of the equations of motion, it is appropriate to compute the Euclidean action with respect to the

ground state that is the extremal black hole in this case [43]. Using similar arguments as in [44,45] it was shown in [4] that there exist extremal black holes with a finite horizon area and so it is expected that the canonical ensemble is well defined.

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