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Optimal paths and costs of adjustment in dynamic DEA models: with application to chilean department stores*

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Abstract In this paper we propose a range of *dynamic* data envelopment analysis (DEA) models which allow information on *costs of adjustment* to be incorporated into the DEA framework. We first specify a basic dynamic DEA model predicated on a number of simplifying assumptions. We then outline a number of extensions to this model to accommodate asymmetric adjustment costs, non-static output quantities, non-static input prices, and non-static costs of adjustment, technological change, quasi-fixed inputs and investment budget constraints. The new dynamic DEA models provide valuable extra information relative to the standard *static* DEA models—they identify an optimal path of adjustment for the input quantities, and provide a measure of the potential cost savings that result from recognising the costs of adjusting input quantities towards the optimal point. The new models are illustrated using data relating to a chain of 35 retail department stores in Chile. The empirical results illustrate the wealth of information that can be derived from these models, and clearly show that static models overstate potential cost savings when adjustment costs are non-zero.

Keywords Cost of adjustment · Dynamic DEA · Path of adjustment

The data envelopment analysis (DEA) method has been applied in a large number of industries over recent decades. The DEA method is a linear programming technique that can be used to measure the relative performance of a group of firms that produce multiple outputs with multiple inputs. Charnes, Cooper, and Rhodes (1978) drew on the work of Farrell (1957) to show how one could use DEA to measure the technical efficiency of each firm

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in the sample[1]—that is, to identify the amount by which inputs could be proportionally reduced without reducing output.[2] Subsequent work has shown how one can also measure scale efficiency, allocative efficiency, cost efficiency and profit efficiency.[3] All of these DEA models identify *efficient targets* – that is, points on a production frontier that each inefficient firm should aim to achieve. However, to our knowledge, none of the existing methods provide information on how the inefficient firm should attempt to reach this optimal point.

For example, consider the case where a clothing factory has a long term contract to produce a particular quantity of clothing, and a DEA analyst tells the manager that the factory should reduce its labour force by 30% and increase the number of sewing machines by 20% to achieve the cost efficient target point (or *optimal point*) determined by the cost efficiency DEA model. How should the manager achieve this? Should the manager attempt to make these changes immediately, or would it be wiser to phase these changes in over a particular period? In the "real world" there are retrenchment costs associated with reducing staff levels; there are various costs associated with the installation of new equipment (e.g. from factory downtime during installation work and staff training costs on new equipment); and there are budget constraints associated with these types of one-off expenditures (i.e. *investment budget constraints*).

In this paper, our aim is to fill this void in the literature. We introduce a range of new DEA models in which the above types of *costs of adjustment* and investment budget constraints are explicitly included as constraints in the linear program. We specify an *adjustment period* (the number of time periods over which the firm can make adjustments); a *discount rate* (an interest rate); and an *appraisal period* (the length of time over which the net present value of the adjustments are evaluated). We then specify an inter-temporal optimisation problem where the firm selects an *optimal point* (set of targets) and an optimal *path of adjustment* (the sequence of input quantities selected in each period) so as to maximise the net present value of profits over the appraisal period. Thus, these new models not only indicate where one should be, but also indicate the steps one should take to get there. We believe that these new *dynamic DEA* models provide valuable information to managers and decision makers over and above the information that existing DEA models provide.

To our knowledge, the dynamic DEA models proposed in this study are the first that attempt to address the issue of inter-temporal decision-making and costs of adjustment in an *ex ante* context. In the past there have been some studies which have looked at the issue of adjustment costs in an *ex post* context. That is, given time series data on a particular industry (or panel data on a group of firms), these previous studies have sought to estimate a dynamic production technology in which adjustment costs are explicitly modelled, and subsequently used to (in part) explain productivity changes over time. In the econometric literature, the key contributions are papers by McLaren and Cooper (1980) and Epstein (1981), who built upon the pioneering theoretical work of Treadway (1970). In the DEA literature, the main contributions are those of Sengupta (1995) and Färe and Grosskopf (1996), plus the recent papers by Nemoto and Goto (1999, 2003).

[1] Similar techniques to those developed by Charnes, Cooper, and Rhodes (1978) had actually been proposed in previous work by Boles (1966) and Afriat (1972) but these papers did not become widely known. See Färe, Grosskopf, and Lovell (1994) for more details.

[2] This is an input-orientated measure of technical efficiency. An alternative output-orientated measure considers the degree to which output could be expanded, with inputs held fixed. If the underlying technology exhibits constant returns to scale (CRS), this output-orientated measure is equal to the inverse of the input-orientated measure.

[3] See Färe, Grosskopf, and Lovell (1985) for a comprehensive treatment of these techniques.

Our models have some similarities with these past models, but there are some important differences. First, these past studies actually estimate adjustment costs as a by-product of the model estimation process (i.e., they implicitly derive the shadow costs of adjustment using historical data), while in our models we explicitly specify the adjustment costs *a priori*, in the same way as one specifies the input prices themselves. Without access to good historical data (which is rarely available), our method is the only viable choice as a management tool. Secondly, and most importantly, our models are designed as *ex ante* management tools, while the studies mentioned above are *ex post* techniques designed to explain past variations in productivity performance in a particular industry over time.

The remainder of this paper is organised into six sections. In Sction 1 we review some of the standard (i.e. static) DEA models that have been widely used as management decision tools in recent decades. In Section 2 we outline a number of concepts and definitions—these are subsequently used in Section 3 where we specify our "basic" dynamic DEA model. Then in Section 4 we outline a number of possible extensions to our basic model, such as asymmetric costs of adjustment and capital investment constraints. In Section 5 we illustrate our methods using data of a chain of Chilean retail department stores, before making some brief concluding comments in Section 6.

1. Review of some standard DEA models

Before we can specify a dynamic DEA model we must first review some of the standard (or *static*) DEA models that are widely used. We begin by presenting the input orientated DEA model of Charnes, Cooper, and Rhodes (1978), for the case where we have a sample of K firms producing J outputs with I inputs:[4]

$$\begin{aligned} \underset{\theta_k,\lambda_k}{\text{Minimise}} \quad & \theta_k \\ \text{subject to:} \quad & \mathbf{X}\lambda_k \leq \theta_k \mathbf{x}_k \\ & \mathbf{Y}\lambda_k \geq \mathbf{y}_k \\ & \lambda_k \geq 0, \end{aligned} \tag{1}$$

where θ_k is the input-orientated technical efficiency measurement for firm k, $\mathbf{X}$ is the $I \times K$ matrix of observed input quantities, $\mathbf{Y}$ is the $J \times K$ matrix of observed output quantities, and the vectors $\mathbf{x}_k$ and $\mathbf{y}_k$ contain the observed input and output quantities (respectively) of firm k.

The DEA model in Eq. (1) measures the technical efficiency of each firm in the sample—that is, it seeks to find the factor, θ_k, by which the k-th firm can shrink its input vector. For example, a technical efficiency score of 0.8 would indicate that the firm could reduce input levels by 20% and still produce the same level of output.

If we also have access to information on the vector of input prices, faced by the firm, we could calculate the cost efficiency of the firm by using the following DEA model, taken from

[4] Note that this is a constant returns to scale (CRS) DEA model. A variable returns to scale (VRS) DEA model is obtained by inserting an additional constraint in Eq. (1) that restricts the elements of the λ_{kt} vector to sum to one.

Färe, Grosskopf, and Lovell (1985).

$$\begin{aligned} \underset{\mathbf{x}_k^*,\lambda_k}{\text{Minimise}} \quad & \mathbf{w}'\mathbf{x}_k^* \\ \text{subject to :} \quad & \mathbf{X}\lambda_k \leq \mathbf{x}_k^* \\ & \mathbf{Y}\lambda_k \geq \mathbf{y}_k \\ & \lambda_k \geq 0, \end{aligned} \tag{2}$$

In this case the decision variables are the input quantities, and we seek to identify that input vector, $\mathbf{x}_k^*$, which the firm can use to produce the given output vector, $\mathbf{y}_k$, at minimum cost, given the observed price vector, $\mathbf{w}$, and the technology defined by the DEA frontier. The cost efficiency score of the k-th firm is calculated as the ratio of the optimal cost to the observed cost. The allocative efficiency score can then be calculated residually as the ratio of the cost efficiency score over the technical efficiency score.

If we also have information on the output price vector, $\mathbf{p}$, faced by the k-th firm, we can identify the profit maximising targets of the firm by changing the objective function in Eq. (1) from "minimise $\mathbf{w}'\mathbf{x}_k^*$" to "maximise $(\mathbf{p}'\mathbf{y}_k^* - \mathbf{w}'\mathbf{x}_k^*)$" and by changing $\mathbf{y}_k$ to $\mathbf{y}_k^*$ on the right-hand-side of the constraints. In this case, both the input and output quantities are decision variables. See Färe, Grosskopf, and Lovell (1985, p. 129) for more on this method.

Finally, as noted earlier, the above static DEA models are defined for a single time period, and do not attempt to identify an appropriate sequence of steps that the firms should take to reach the optimum point that has been identified. Hence, in subsequent sections, we propose new dynamic DEA models which are generalisations of these standard (static) models.

2. Some concepts and definitions

The following concepts are useful for the definition and the solution of the dynamic DEA models examined in this paper.

Adjustment: the change in the input quantity vector, $\mathbf{x}_{kt}$, of firm k that is made at the start of period $t = 1, 2, \ldots, T$. The adjustment may be an increase, $\mathbf{x}_{kt}^+$, or a decrease, $\mathbf{x}_{kt}^-$, in input quantities. When it is not necessary to distinguish whether the adjustment is an increase or decrease, the generic notation, $\mathbf{x}_{kt}^a$, for adjustment is used. The adjustments are decision variables in the DEA model.

Cost of adjustment : the cost incurred by a firm when it adjusts (changes) the quantity of some input, such as labour. The cost of adjustment of an input differs from its price, and is specific for each input and to each firm. When the cost of increasing an input differs from the cost of decreasing it we say the costs of adjustment are *asymmetric*. The concept of a cost of adjustment has a long history in economics—see, for example, Treadway (1970).

Dynamic DEA: a DEA model that involves optimisation over two or more time periods, where period-to-period variations in input quantities incur costs of adjustment.

Path of adjustment: the optimal sequence of input quantity vectors, $\mathbf{x}_{kt}$, $t = 1, 2, \ldots,$ that an inefficient firm selects in order to move from its initial input quantity vector to the target (or optimal) input vector. The path of adjustment is optimal in the sense that it maximises the net present value of the profit of the firm, discounted at a constant compound rate, over the specified time horizon.

Period: the unit of time measurement. It is assumed that within periods all variables have constant values; nonetheless these values may change from one period to another. Once the span of time involved is defined, the period is irrelevant for the derivation and statement of the mathematical expressions of the model. In the application involving Chilean department stores discussed later in this paper, one period is taken to be six months unless otherwise specified.

Profit: the gross income less the cost of inputs and less the costs associated with adjusting inputs to their target values.[5]

Adjustment period: the number of periods that a firm allows itself to achieve its targets. Within this adjustment period, denoted t_a, the initial input quantity vector, $\mathbf{x}_{k0}$, is transformed into the target input quantity vector, $\mathbf{x}_k^*$. The adjustment period is constrained to a prefixed maximum number of periods, t_a^*.

Appraisal period: the predetermined number of periods that management considers for the economic evaluation of each specific investment project. For the purpose of this work, the adjustments that each firm makes to the input quantity vector are viewed as a specific investment project to be evaluated. Economic evaluation involves computing the net present value of the total profit of each project over the appraisal period. The appraisal period, denoted T, may be equal to or larger than the adjustment period. If the appraisal period is longer than the adjustment period, the firm capitalises along $T - t_a$ extra periods the cost savings derived from the reduction in the input quantities.

3. A basic dynamic DEA model

In this basic dynamic DEA model[6] we assume that the firm is required to produce a fixed output vector, $\mathbf{y}_k$. The problem for firm k is to choose the time sequence of input quantity adjustment vectors, $\mathbf{x}_{kt}^a$, $(t = 1, 2, \ldots, t_a)$, which maximises the net present value of profit. This time sequence of adjustments of inputs transforms the initial input quantity vector, $\mathbf{x}_{k0}$, into the target input quantity vector, $\mathbf{x}_k^*$. The adjustments are performed within the specified adjustment period, t_a^*. The time sequence of vectors, $\mathbf{x}_{kt}$, $t = 1, 2, \ldots, t_{a,}$ which maximises the net present value of profit is the *optimal path of adjustment.*

For modelling purposes, it is assumed that all variables have constant values within each period, although they may change value from one period to the next. For evaluation purposes, we assume that adjustment costs are incurred at the start of each period while gross income is received and input costs are incurred at the end of each period. Also, consistent with the assumption that values are constant within each period, we neglect transient values of variables. This means that variables behave as expected from the start of the period—any set up or start up times are negligible.

[5] Note that we are implicitly assuming that the manager maximises utility by maximising profit. If some other factors enter into the managers' utility function (such as a sense of loyalty to long serving employees) then the model formulation would need to be adjusted to reflect this.

[6] This is called a *basic* model because we make a number of simplifying assumptions in this model. For example, we assume that there is no technical change and no change in the required output quantity over the investment appraisal period. These and other assumptions are relaxed in later sections of this paper.

For firm k, at the end of any period t, the cost of inputs is $\mathbf{w}'\mathbf{x}_{kt}$, where $\mathbf{w}$ is the vector of constant input prices. The vector of adjustments to the input quantity vector at period t, $\mathbf{x}_{kt}^{a}$, has a total cost of adjustment of $\mathbf{w}^{a'}\mathbf{x}_{kt}^{a}$, where $\mathbf{w}^{a}$ is the vector of costs of adjustment. At the end of each period, the gross income of firm k is $\mathbf{p}'\mathbf{y}_k$, where $\mathbf{y}_k$ is a fixed and known output vector and $\mathbf{p}$ is a vector of constant output prices.

In this work, we assume that a firm is of a size such that it cannot influence the markets for its products. As a buyer, the firm regards the price of inputs as given, for the same reason. These firms are known as *price-taking firms*(e.g. see Chambers, 1988, p. 50).

With these revenues and costs, the profit, π_{kt}, of firm k in period $t$is calculated as

$$\pi_{kt} = \mathbf{p}'\mathbf{y}_k - \mathbf{w}'\mathbf{x}_{kt} - \mathbf{w}^{a'}\mathbf{x}_{kt}^{a}$$

Hence, the net present value of the profit of firm $k$over the evaluation period is:

$$\pi_k = \sum_{t=1}^{T} \left[s_t(\mathbf{p}'\mathbf{y}_k - \mathbf{w}'\mathbf{x}_{kt}) - s_{t-1}\mathbf{w}^{a'}\mathbf{x}_{kt}^{a} \right] \tag{3}$$

where s_t is the present value factor from period t, as defined below.

The mathematical formulation of the dynamic DEA problem of firm k is:

$$\underset{\mathbf{x}_{kt}^{a},\lambda_{kt}}{\text{Maximise}} \quad \pi_k = \sum_{t=1}^{T} \left[s_t(\mathbf{p}'\mathbf{y}_k - \mathbf{w}'\mathbf{x}_{kt}) - s_{t-1}\mathbf{w}^{a'}\mathbf{x}_{kt}^{a} \right] \tag{4}$$

$$\begin{array}{lll}
\text{subject to} & & \\
\left.\begin{array}{l} \mathbf{Y}\lambda_{kt} \geq \mathbf{y}_k \\ \mathbf{X}\lambda_{kt} \leq \mathbf{x}_{kt} \end{array}\right\} & 1 \leq t \leq T & \text{(technology constraint)} \\
\mathbf{x}_{kt} = \mathbf{x}_{k,t-1} + \mathbf{x}_{kt}^{a} & & \text{(transition equation)} \\
\mathbf{w}^{a'}\mathbf{x}_{kt}^{a} \leq b_{kt} & & \text{(budget constraint)} \\
s_t = (1 + r/100)^{-t} & & \text{(present value factor)} \\
\lambda_{kt}, \mathbf{x}_{kt}, \mathbf{x}_k^{*}, \mathbf{y}_k \geq 0 & & \text{(non}-\text{negative variables)}
\end{array}$$

Similar LP problems must be written for each one of the K firms.

Model (4) has $\mathbf{x}_{kt}^{a}$ and λ_{kt} as optimisation variables and solves, in one step, the identification of targets and the definition of the optimal path of adjustment. Hereafter, we refer to model (4) as the *basic dynamic DEA model*. Although the solution to problem (4) identifies period to period changes in the input quantity vector of the firm under study, $\mathbf{x}_{kt}$, the observed data that define the boundary of the technology remains unchanged.

Model (4) reduces to the static cost minimisation problem in Eq. (1) for the case when $T = 1$, and constant output vector $\mathbf{y}_{k.}$With that condition, the present value of profit is the value for one period, the input quantity vector corresponds to the target input quantity vector, and, since there are no adjustments, the cost of adjustment is zero. For this reason, the standard (static) profit maximisation problem discussed in Section 1 is a particular form of model (4).

Model (4) presents two issues that deserve special attention. First, the vector of weights, λ_{kt}**,** is permitted to change from period to period. At first glance, this may appear unnecessary. However, when it is noted that the input mix of the firm can change from period to period during the adjustment process, it is clear that the λ_{kt} weights must be permitted to change so as to ensure that the vectors of inputs and outputs of the k-th firm (that are proposed by the solution of the dynamic DEA problem) remain within the boundary of the production technology in each and every period.

The second issue is that the input quantities are not decision variables in Eq. (4), as they were in the standard static cost minimisation problem in Eq. (2). Rather, the input quantity adjustment vectors, $\mathbf{x}^a_{kt}$, are the decision variables in this case (in addition to the λ_{kt}); the input quantity vector at period t is the result of decisions on the adjustment of inputs, not a decision in itself. The *transition equation*, $\mathbf{x}_{kt} = \mathbf{x}_{k,t-1} + \mathbf{x}^a_{kt}$, defines the input quantity vector in adjustment period t in terms of the input quantity vector in the previous adjustment period and the input quantity adjustment vector in period t. In the basic DEA model in Eq. (4), the transition equations are constraints. This consideration, together with the observation that the λ_{kt} may vary from period to period, permits us to visualise this dynamic DEA model as a time sequence of fundamental DEA models, linked by a common objective function and a sequence of transition equations.

4. Extensions to the basic dynamic DEA model

In this section we consider some extensions to the basic dynamic DEA model defined in the previous section. Among other things, we consider asymmetric adjustment costs, non-static output quantities, non-static input prices, non-static costs of adjustment, technological change, quasi-fixed inputs and investment budget constraints.

4.1. Asymmetric costs of adjustment

It is quite possible that the costs associated with increasing the quantity of a particular input will differ from the cost associated with decreasing it. For example, retrenchment costs may differ from the training costs associated with putting on new staff.

In terms of the model in Eq. (4), allowing for asymmetric costs of adjustment requires modification of the objective function, as well as the budget and transition constraints. When making these modifications we must be careful to distinguish between upward and downward adjustments to input quantities, and the costs associated with them. Let $\mathbf{x}^+_{kt}$ and $\mathbf{x}^-_{kt}$ denote increases and decreases in inputs, and $\mathbf{w}^+$ and $\mathbf{w}^-$ denote the costs of increasing and decreasing inputs, respectively. The modified mathematical expressions are:

$$\underset{\mathbf{x}^+_{kt},\mathbf{x}^-_{kt},\lambda_{kt}}{\text{Maximise}} \quad \pi_k = \sum_{t=1}^{T} [s_t(\mathbf{p}'\mathbf{y}_k - \mathbf{w}'\mathbf{x}_{kt}) - s_{t-1}(\mathbf{w}^{+'}\mathbf{x}^+_{kt} + \mathbf{w}^{-'}\mathbf{x}^-_{kt})] \tag{5}$$

$$\mathbf{w}^{+'}\mathbf{x}^+_{kt} + \mathbf{w}^{-'}\mathbf{x}^-_{kt} \le b_{kt} \qquad \text{(Budget constraint)}$$

$$\mathbf{x}_{kt} = \mathbf{x}_{k,t-1} + \mathbf{x}^+_{kt} - \mathbf{x}^-_{kt} \qquad \text{(Transition equation)}$$

4.2. Appraisal period longer than adjustment period

The basic dynamic DEA model assumes there is no difference between the appraisal period and the adjustment period. However, in some situations this may be unrealistic.

If we view the adjustment of input quantities as an investment project, then the adjustment period represents the number of periods during which investments are made. Each investment is a period-to-period adjustment to input quantities. The number of periods that a firm effectively uses to perform the adjustment, t_a, is constrained by the maximum (pre-specified) time of adjustment, t_a^*.

As previously stated, the adjustment period is the predetermined number of periods that management considers for the economic evaluation of each specific investment project. In most cases, the number of periods used for economic evaluation of an investment project is larger than the number of periods over which the investment is made.

Because the savings derived from input adjustments are evaluated over the appraisal period, the present value of savings may be larger than the present value of costs of adjustment evaluated only over the adjustment period. Thus, increasing the length of the appraisal period has the potential to increase the net present value of the cost savings and hence the profits.

This extension to the basic model involves modifying the budget constraint and transition equation, making explicit the period of time for which they are valid. We also include the constraint that the time taken to adjust inputs has to be less than or equal to the maximum adjustment period. The modified mathematical expressions are:

$$\begin{aligned} &\mathbf{w}^{+'}\mathbf{x}_{kt}^{+} + \mathbf{w}^{-'}\mathbf{x}_{kt}^{-} \leq b_{kt}, t = 1, 2, \ldots, t_a \quad \text{(Budget constraint)} \\ &\mathbf{x}_{kt} = \mathbf{x}_{k,t-1} + \mathbf{x}_{kt}^{+} - \mathbf{x}_{kt}^{-}, t = 1, 2, \ldots, t_a \quad \text{(Transition equation)} \\ &t_a \leq t_a^* \end{aligned} \tag{6}$$

4.3. Dynamic expectations

The basic model specifies that output quantities do not change throughout the appraisal period. In this section, we present a dynamic DEA model with dynamic (i.e. non-static) outputs. The term *dynamic output* refers to the case when the expected output quantities of the firm may change from period to period.[7] These future period-to-period target output quantity vectors may be estimated by experts or may be set by managers.

The existence of dynamic outputs implies that as long as the output quantity vector changes from period to period, the input quantity vector may also most likely need to change from period to period. Thus, with dynamic output quantity vectors, when output quantities change in each period of the appraisal period, there are no practical reasons for differentiating the appraisal period and the time of adjustment.

This extension to the basic DEA model involves modifying the objective function and the output quantity vector on the right hand side of the boundary constraint. The modified mathematical expressions (including asymmetric costs of adjustment) are presented in Eq. (7), where the subscript t on the output vector indicates that it may change from period to period. The mathematical expressions in Eq. (7) also allow for changes in input and output prices from one period to the next. This will be useful if price relativities are expected to change, but is not important if all prices are expected to increase by a common inflation multiple.[8]

$$\underset{\mathbf{x}_{kt}^{+},\mathbf{x}_{kt}^{-},\lambda_{kt}}{\text{Maximise}} \quad \pi_k = \sum_{t=1}^{T} [s_t(\mathbf{p}'_t\mathbf{y}_{kt} - \mathbf{w}'_t\mathbf{x}_{kt}) - s_{t-1}(\mathbf{w}_t^{+'}\mathbf{x}_{kt}^{+} + \mathbf{w}_t^{-'}\mathbf{x}_{kt}^{-})] \tag{7}$$

[7] Note that here we assume that the frontier does not shift over time. That is, no technical change. We relax this assumption in later sections of this paper.

[8] Note that one would normally specify real prices, and hence a real discount rate in these models. There is no advantage to be gained from using nominal prices and introducing the additional complexity of forecasting inflation rates.

$$\left.\begin{array}{l}\mathbf{Y}\lambda_{kt} \geq \mathbf{y}_{kt} \\ \mathbf{X}\lambda_{kt} \leq \mathbf{x}_{kt}\end{array}\right\} \quad 1 \leq t \leq T \qquad \text{(Boundary of technology)}$$

$$\mathbf{w}_t^{+'}\mathbf{x}_{kt}^{+} + \mathbf{w}_t^{-'}\mathbf{x}_{kt}^{-} \leq b_{kt} \qquad \text{(Budget constraint)}$$

$$\mathbf{x}_{kt} = \mathbf{x}_{k,t-1} + \mathbf{x}_{kt}^{+} - \mathbf{x}_{kt}^{-},\ t = 1, 2, \ldots, T \qquad \text{(Transition equation)}$$

4.4. Quasi-fixed input variables

Quasi-fixed or non-discretionary variables are variables that may not be modified at management's discretion in a short time horizon. The acres of land in a farm, the location of a firm, and the number of competitors close to a firm are examples of quasi-fixed variables. The terms quasi-fixed and non-discretionary are generally used interchangeably.

Staat (1999, p. 42) discusses DEA-techniques for modelling continuous and categorical non-discretionary variables. Staat (1999) presents the model developed by Golany and Roll (1993, p. 423f). For the purpose of this work, we adapt the model of Banker and Morey (1986) that Cooper, Seiford and Tone (2000, p. 63) present for technical efficiency measurement. The adapted model with discretionary outputs and discretionary and non-discretionary inputs is model (8):

$$\underset{\mathbf{x}_{kt}^{+},\mathbf{x}_{kt}^{-},\lambda_{kt}}{\text{Maximise}} \quad \pi_k = \sum_{t=1}^{T} [s_t(\mathbf{p}'_t\mathbf{y}_{kt} - \mathbf{w}'_t\mathbf{x}_{kt}) - s_{t-1}(\mathbf{w}_t^{+'}\mathbf{x}_{kt}^{+} + \mathbf{w}_t^{-'}\mathbf{x}_{kt}^{-})] \tag{8}$$

subject to

$$\mathbf{w}_t^{+'}\mathbf{x}_{kt}^{+} + \mathbf{w}_t^{-'}\mathbf{x}_{kt}^{-} \leq b_{kt},\ t = 1, 2, \ldots, t_a \qquad \text{(Budget constraint)}$$

$$\mathbf{x}_{kt} = \mathbf{x}_{k,t-1} + \mathbf{x}_{kt}^{+} - \mathbf{x}_{kt}^{-}, \quad t = 1, 2, \ldots, t_a \qquad \text{(Transition equation)}$$

$$\left.\begin{array}{l}\mathbf{Y}\lambda_{kt} \geq \mathbf{y}_{kt} \\ \mathbf{X}\lambda_{kt} \leq \mathbf{x}_{kt} \\ \mathbf{Z}\lambda_{kt} = \mathbf{z}_k\end{array}\right\} \quad 1 \leq t \leq T \qquad \text{(Boundary of technology)}$$

$$t_a \leq t_a^* \qquad \text{(Time to adjust inputs is lower than or equal to adjustment time);}$$

all variables are non-negative;
where $\mathbf{Z}$ is an $N \times K$ matrix of non-discretionary input quantities; $\mathbf{z}_k$is the $N \times 1$ vector of non-discretionary input quantities of firm k; N is the number of non-discretionary inputs; and K is the number of observed firms.

4.5. Incorporation of a capital investment constraint

In addition to non-discretionary variables, we now present capital investment as a special input variable, which may be modified at management's discretion, but under prefixed constraints. We refer to such variables as *constrained discretionary variables*. The constraints to capital investment may come from a specific investment policy that is derived from a strategic planning process or from short-term capital allocation priorities. Two constraints are considered here: fixed assets, $K(t)$, and fixed capital investment $I(t)$ at period t.

Consider the capital investment ratio presented by Luh and Stefanou (1996, p. 993),

$$\dot{K}(t) = I(t) - \delta K(t); K(0) = k. \tag{9}$$

The discrete-time version of this first-order ordinary differential equation is:

$$K(t) = K(t-1) + I(t) - \delta K(t-1); K(0) = k, \tag{10}$$

where $K(t)$ is the fixed asset at the start of period t; $I(t)$ is the capital investment in period t; δ is the rate of depreciation of assets; and $K(0) = k$ is the amount of capital at the time that the decisions are made.

Equation (10) is similar to the transition equations of the basic DEA model (4). The only difference is that the adjustments to these transition equations are *either* increases *or* decreases, while the adjustments for the constrained discretionary variables may be *both* increases (with investment) *and* decreases (with depreciation).

Taking this perspective, the constrained discretionary variables may be considered as discretionary variables with a special transition equation. To be consistent with previous definitions of variables, let $\mathbf{q}_{kt}$ be the $M \times 1$ constrained discretionary input quantity vector of firm k in period t (instead of $K(t)$); M the number of constrained discretionary inputs; $\bar{\mathbf{q}}_{kt}$ be the maximum of the constrained discretionary input quantity vector for firm k in period t; $\mathbf{c}_{kt}$ be the capital investment vector of firm k in period t(instead of $I(t)$); and $\bar{\mathbf{c}}_{kt}$ be the maximum of the capital investment vector for firm k in period t. The cost per period of $\mathbf{q}_{kt}$ is denoted $\mathbf{v}_{kt}$.

Finally, the inclusion of constrained discretionary variables in the DEA model for optimal paths of adjustment results in the following:

$$\begin{aligned}
&\mathbf{w}_t^{+'}\mathbf{x}_{kt}^{+} + \mathbf{w}_t^{-'}\mathbf{x}_{kt}^{-} \le b_{kt}, \quad t = 1, 2, \ldots, t_a && \text{(Budget constraint)}\\
&\mathbf{x}_{kt} = \mathbf{x}_{k,t-1} + \mathbf{x}_{kt}^{+} - \mathbf{x}_{kt}^{-}, \quad t = 1, 2, \ldots, t_a && \text{(Transition equation)}\\
&\mathbf{q}_{kt} = \mathbf{q}_{k,t-1} + \mathbf{c}_{kt} - \delta\mathbf{q}_{k,t-1}\\
&\mathbf{q}_{kt} \le \bar{\mathbf{q}}_{kt}\\
&\mathbf{c}_{kt} \le \bar{\mathbf{c}}_{kt}\\
&\left.\begin{aligned}\mathbf{Y}\lambda_{kt} &\ge \mathbf{y}_{kt}\\ \mathbf{X}\lambda_{kt} &\le \mathbf{x}_{kt}\\ \mathbf{Z}\lambda_{kt} &= \mathbf{z}_k\\ \mathbf{Q}\lambda_{kt} &\le \mathbf{q}_{kt}\end{aligned}\right\} \quad 1 \le t \le T && \text{(Boundary of technology)}\\
&t_a \le t_a^*
\end{aligned}$$

where $\mathbf{Q}$ is the matrix of constrained discretionary input quantities.

4.6. Technical change

Our basic dynamic DEA model assumes that the location of the frontier does not shift over the appraisal period. This is likely to be a reasonable assumption in most industries when the appraisal period is short (e.g. less than five years). However, in situations where the appraisal period is longer and/or the industry normally experiences fast technical change (e.g. in the telecommunications industry), this assumption of no technical change may not be reasonable. In practice, it would be quite difficult to estimate the degree to which the frontier is likely to be shifted by technical change, other than to extrapolate recent observed shifts in the estimated DEA frontier for that industry. However, for completeness we make note of this option. In terms of the above LP's, the only required change is to put t-subscripts on the matrices of inputs and outputs, $\mathbf{Y}_t$ and $\mathbf{X}_t$, which are used to define the boundary of the technology in each period.

5. An application to Chilean department stores

In this section we illustrate these new dynamic DEA models using data on a chain of 35 Chilean retail department stores. This data is ideally suited to this type of model because we were able to obtain detailed information on outputs, input quantities and input prices from the company accounts, and we were also able to elicit accurate and realistic measures of per unit adjustment costs through direct discussions with management.

5.1. Data

Quantity and price data were extracted from the accounting information of the 35 stores for the years 2000 and 2001.[9] The labour-related costs of adjustment are based upon Chilean labour regulations. Non-labour adjustment costs were estimated by senior management. Under a confidentiality agreement with the firm, data are encrypted by simple scaling transformations. The scaling factor is specific to each kind of variable. This simple encryption of data does not change the optimal value of the decision variables when transformed back to the original scale. After being encrypted, the monetary data are in "Firm's Monetary Units" (F$), and the store floor data are in "Firm's Surface Units" (FSU). For all purposes, *period* refers to a time period of six months.

Output

For performance evaluation of the stores, the firm's senior managers consider *gross sales* to be the most sensible measure of the output of a store. This measure is deemed to be an appropriate index of output, given that the stores sell a similar range of products at similar prices.

Inputs

The management proposed five inputs as being most relevant for branch performance evaluation. Four of these inputs are discretionary variables and a fifth is a non-discretionary variable. These five variables are:

(1) *Salesperson labour*—measured in thousands of hours per period.
(2) *Cashier labour*—measured in thousands of hours per period.
(3) *Sales and administration expenses*—measured in F$ per period.
(4) *Marketing expenses*—measured in F$ per period.
(5) *Store floor surface*—measured in FSU. This is a non-discretionary variable.[10]

[9] Data from these four six-month periods were averaged to reduce the impact of measurement error and random fluctuations upon the starting values of the output and input quantities and prices in the empirical exercise.

[10] Further information on the data can be obtained from de Mateo (2003).

Lengths of adjustment and appraisal periods

The maximum length of the adjustment period was set at four periods. The length of the appraisal period (for the economic evaluation of the adjustment/investment project) was set at eight periods, which is equivalent to four years.

Rate of discount

We consider that an appropriate (real) rate of discount is 9.0 per cent per period for this type of investment project.

Budget constraints

We assume that no store could spend on this project more than three per cent of its gross profit in the base period. This gross profit is evaluated as the residual of output revenue and the cost of inputs.

5.2. Results

In this section we present empirical results for two different models:

Model 1 = our basic dynamic DEA model, plus asymmetric costs of adjustment; appraisal period longer than adjustment period; quasi-fixed inputs; and a capital investment constraint.

Model 2 = Model 1, plus dynamic output quantities, input prices and adjustment costs.[11]

The volume of information produced by these models is quite large. In order to keep this results section to a manageable size, we will present brief results for all 35 stores and detailed results for one store only, store 232.

Model 1

Relevant data for store 232 is presented in Tables 1 and 2. It is difficult to provide too much discussion of these data, given that they have been scaled for confidentiality reasons. However, we can observe that the ratio of adjustment cost to price is larger for salesperson labour than for cashier labour, reflecting the higher training costs involved and the more permanent nature of salesperson labour. We should also note that our model assumes the same cost of adjustment associated with retrenchment of each marginal employee. In some cases it may be more reasonable to specify a model where this is not constant. For example, 5% of the labour force may leave through natural attrition per six-month period and hence involve zero adjustment cost. The next 10% of labour reduction could be achieved by paying out short term contract staff and hence involve less adjustment costs than retrenching members of the more permanent employee group. Hence, one could consider that a step function of

[11] It should be noted that the output quantities are not permitted to be decision variables in either of these models. It was our view that the output of these stores is largely determined by the size of the local population, and hence the store managers have little discretion over this. Thus the empirical models produced in this section are actually the cost minimisation special case of the profit maximisation model discussed earlier. If we were to consider an alternative industry, such as small scale farming, then the assumption of discretionary output quantities would be more appropriate.

Table 1 Initial data for store 232 for Model 1

Variable	Value
Gross sales (F$)	1,987.1
Salesperson labour (khrs)	4.294
Cashier labour (khrs)	2.462
Sales & General exp. (F$)	75.22
Marketing exp. (F$)	18.03
Store area (FSU)	7.31

Table 2 Prices of inputs and costs of adjustment for store 232 for Model 1

Variable	Price	Increase	Decrease
Sales person labour (F$/khrs)	11.86	4.240	8.604
Cashier labour (F$/khrs)	5.071	1.034	2.188
Sales & General exp. (F$)	1.00	0	2.04
Marketing exp. (F$)	1.00	0	0.14

Table 3 Model 1 optimal path of adjustment for store 232

Period	**Initial**	**1**	**2**	**3**	**4**
Sales person labour (khrs)	4.924	4.787	4.787	4.787	4.787
Cashier Labour (khrs)	2.462	1.219	1.219	0.993	0.993
Sales & General exp. (F$)	75.22	50.32	23.51	19.37	19.37
Marketing exp. (F$)	18.03	22.48	22.48	24.58	24.58
Store area (FSU)	7.31	7.31	7.31	7.31	7.31
Cost of inputs (F$)	164.13	135.70	108.95	105.76	105.76
Cost of adjustment (F$)	0	54.69	54.69	8.95	0
Cost efficiency	0.644	0.7682	0.971	1.0	1.0

adjustment costs may apply. However, in this empirical illustration we have decided to keep the model simple, and specify a constant adjustment cost for each marginal unit.

The optimal path of adjustment for store 232 is presented in Table 3. In the column labelled "Initial" we list the initial input and output quantities from Table 1, along with the total cost of inputs in that period (F$164.13), the costs of adjustment for the period (which are zero by definition), and the cost efficiency score, which is 0.644. This cost efficiency score indicates that this firm could reduce its input costs by 35.6% and still produce the same output. However, this is a traditional (i.e. static) cost efficiency score, which does not take into account costs of adjustment.

The next column in Table 3 lists the input quantities after the first round of adjustments proposed by the solution of the dynamic DEA problem. It is interesting to note that the LP has chosen to reduce the quantities of three of the variable inputs (sales and general expenses, sales person labour and cashier labour), while increasing the fourth variable input (marketing expenses). This indicates that, for this particular store, the initial level of marketing expenses was *below* the cost-minimising level. We also note that the fixed input variable (store area) remains fixed at 7.31 units, as is required.

The cost efficiency score increases from 0.644 in the initial period to 0.768 after the first period of adjustment, indicating that the store is moving towards its optimal point. Additional adjustments are made in periods two and three, with the optimal point achieved in period three. Thus there are no reported adjustments in period four.

Table 4 Summary results of the optimal paths of adjustment for Model 1*

Store	Cost of Inputs (F$)		Cost of adjustment (F$)	Net saving (F$)	Net saving as % of cost of initial inputs
	Initial conditions	Optimal path			
202	1,062	630.1	46.78	383.1	36.07
204	830.2	501.1	45.54	283.56	34.16
211	938.3	804.1	25.0	109.2	11.64
224	910.2	553.0	47.21	310.0	34.05
225	1054	890.2	39.10	124.7	11.83
226	1091	803.5	38.21	249. 59	22.88
232	1,073	779.7	112.4	180.9	16.86
234	1,232	1,165	13.6	54.3	4.33
235	916.6	823.5	19.2	73.9	8.06
236	1,040	1,040	–	–	–
237	889.2	889.2	–	–	–
238	961.4	590.1	50.1	321.2	33.41
239	972.0	892.2	17.4	79.8	8.21
240	1,127	860.9	56.8	209.3	18.57
242	905.8	696.6	41.8	167.4	18.48
243	821.2	735.3	23.0	62.9	7.66
244	1,356	1336	13.4	6.6	0.49
245	1,170	1011	29.8	129.2	11.04
246	897.6	879.0	6.3	12.3	1.37
247	887.8	839.9	11.2	36.7	4.13
248	1,042	906.4	28.9	135.6	13.01
250	976.6	876.3	20.6	79.7	8.16
251	810.5	777.1	20.3	13.1	1.62
252	987.3	810.3	28.9	148.1	15.00
254	892.5	833.2	15.0	44.3	4.96
256	1,133	822.3	60.8	249.9	22.06
257	965.3	777.9	30.9	156.5	16.21
258	1,011	835.3	29.6	146.1	14.45
259	1,129	995.2	22.7	111.1	9.84
260	1,050	1001	7.60	41.40	3.94
262	846.2	636.6	27.2	182.4	21.26
263	1,011	980.8	12.9	17.3	1.71
264	1,007	973.1	12.9	21.0	2.09
265	933.6	933.6	–	–	–
266	720.1	475.6	34.6	209.9	29.15
Total	34,650	29,355	990	4,101	11.84

* Costs and savings are present value calculations over the eight periods.

Summary results for all 35 stores are reported in Table 4. The reported cost of inputs at *initial conditions* indicates the present value of the cost of inputs (evaluated over the eight periods) if the quantities used in the initial period are also used in each one of the subsequent eight periods. The reported cost of inputs of the *optimal path* is the present value of the cost of inputs (evaluated over the eight periods) if the optimal path quantities are used. The reported cost of adjustment is the present value (evaluated over the eight periods) of the costs of adjustment of the inputs. The reported net saving is the difference between the present

Table 5 Amended information for store 232 for Model 2

Variable	Period: Output (F$)	Initial 1,987.10	1 2,026.84	2 2,046.71	3 2,066.58	4 2,086.46
Salesperson	Price	11.86	11.95	12.03	12.25	12.39
Labour	Increase	20.80	20.97	21.09	21.49	21.74
	Decrease	42.20	42.54	42.79	43.59	4.01
Cashier	Price	5.071	5.10	5.14	5.23	5.27
Labour	Increase	5.071	5.10	5.14	5.23	5.27
	Decrease	10.73	10.79	10.87	11.07	11.16
Sales &	Price	1.0	1.0	1.0	1.0	1.0
general exp.	Increase	0	0	0	0	0
	Decrease	2.04	2.06	2.091	2.105	2.132
Marketing	Price	1.0	1.0	1.0	1.0	1.0
exp.	Increase	0	0	0	0	0
	Decrease	0.140	0.1417	0.1470	0.1445	0.1463

Table 6 Model 2 optimal path of adjustment for store 232

Period	Initial	1	2	3	4
Output (F$)	1,987.10	2,026.84	2,046.71	2,066.58	2,086.46
Sales person labour (khrs)	4.924	4.924	4.924	4.984	5.033
Cashier labour (khrs)	2.462	1.837	1.228	1.043	1.043
Sales & General exp. (F$)	75.223	49.37	23.86	20.32	20.32
Marketing exp. (F$)	18.03	20.85	23.52	25.54	25.86
Store area (FSU)	7.31	7.31	7.31	7.31	7.31
Cost of inputs (F$)	164.13	139.25	114.09	113.85	116.14
Cost of adjustment (F$)	0	54.69	54.69	8.14	0.219
Cost efficiency	0.644	0.7844	0.9751	0.9990	1.000

value of the cost of inputs at initial conditions and the sum of the present value of the cost of inputs of the optimal path of adjustment and the present value of the cost of adjustment.

From Table 4, we observe that the present value of total net saving of the 35 stores is F$4,101. This figure is difficult to interpret, given the data have been scaled for confidentiality purposes. However, we see that this equates to a saving of 11.84 per cent of the cost of inputs at the initial conditions, which indicates that the managers could achieve significant savings if the proposed optimal paths are followed. The potential savings differ significantly from store to store, ranging from zero per cent for the three initially fully efficient stores (236, 237, and 265), up to a 36.07 per cent reduction in costs for Store 202.

It is of interest to note that the percentage of potential cost savings reported for store 232 in Table 4 is 16.86 per cent. This is less than the potential cost savings of 35.6 per cent indicated by the static cost efficiency score of 0.644 reported in Table 3 for store 232. This illustrates an important point—a *static* efficiency measure will tend to overstate the potential savings when adjustment costs are non-zero (which is almost always the case).

Model 2

We now provide an illustration of how things can change when we allow for the (arguably) more realistic situation where we expect output quantities and (real) prices to change over time. The changes that we specify for store 232 (based upon the expert judgment of managers) are listed in Table 5, and the new optimal path of adjustment is presented in Table 6. The first thing we note is that the projected increase in output results in a reduction in the amounts by which the input quantities need to be reduced. In particular, for the case of salesperson hours, the optimal quantity now *increases* over time (it decreased in Model 1), reflecting the extra sales persons needed to deal with the expected increase in sales volume. This is a logical result, which illustrates the dangers in the use of static expectations in dynamic optimisation models.

It is interesting to observe that total costs of adjustment have increased slightly in Model 2 compared with Model 1. One would normally expect that the reduction in required input adjustments (in Model 2) would result in lower costs of adjustment. However, this does not occur here because we have also allowed the per-unit costs of adjustment to increase over time in Model 2. Hence the two effects tend to cancel each other out in this instance.

Two additional points should be made before we conclude this results section. First, the final optimal point in these DEA models need not be on the DEA frontier. The frontier simply defines the outer boundary of what is technically feasible. If there are no adjustment costs (or other non-traditional constraints) then the firm will clearly move to the optimal *static* cost minimising point on the frontier (i.e. where cost efficiency equals one) in one step. This is the standard Fare et al. (1985) cost-minimising DEA model. However, with adjustment costs and extra constraints included in the model, there is no need for the final optimal point to be actually on the frontier. This will only occur if it is an optimal thing to do, in a NPV sense. In our Model 1 example all firms reached the frontier during the specified period. However, in Model 2 this was not always the case because many parameters were assumed to change period by period, which is arguably a more accurate reflection of the real world.

A second, and related point to stress is that if these methods are to be adopted by management, it would make sense for the managers to re-run the dynamic DEA model in each period, once new information comes available in each period. In this situation, only the first period adjustment would actually be implemented in each optimal path. Thus the store would be forever heading towards an optimal point, but unlikely to ever actually reach the desired target, unless prices and output quantities do not change over a number of periods, or if the optimal point could be reached in a single period of adjustment.

6. Conclusions

In this paper we began with a review of the most commonly used standard (static) DEA models, and observed that they provide valuable information on the optimal point that a firm should attempt to reach. However, we also noted that they do not provide any information on the path that the firm should take in order to reach this optimal point. To address this issue, we propose a range of *dynamic* data envelopment analysis (DEA) models which allow us to explicitly incorporate information on *costs of adjustment* into the DEA framework. This allows us to not only identify the optimal path of adjustment, but also provides information on the potential cost savings that can be achieved when costs of adjustment are explicitly recognised.

We first specified a basic dynamic DEA model involving a number of simplifying assumptions. This was done so as to convey the basic nature of the model, without clouding the issue with too many complexities. We then outlined a number of extensions to this basic model to allow us to deal with various issues including asymmetric adjustment costs, non-static output quantities, non-static input prices, and non-static costs of adjustment, technological change, quasi-fixed inputs and investment budget constraints.

Some of the new dynamic DEA models were then illustrated using data relating to a chain of 35 retail department stores in Chile. The empirical results obtained illustrated the wealth of information that can be derived from these models, and clearly show that static models overstate potential cost savings when adjustment costs are non-zero. For example, in the case of store 232 (which was chosen for detailed analysis) we found that the static model suggested annual cost savings of 35.60 per cent, while the dynamic model suggested more modest cost savings of 16.86 per cent when costs of adjustment were taken into account.

Overall, we found that the 35 stores could save 11.84 per cent of total input costs if they were to adopt the proposed optimal paths of adjustment. This indicates that these methods have the potential to provide valuable information to the managers of this business.

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