Asymptotically (anti-) de Sitter black holes and wormholes with a self-interacting scalar field in four dimensions

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The aim of this paper is to report on the existence of a wide variety of exact solutions, ranging from black holes to wormholes, when a conformally coupled scalar field with a self-interacting potential containing a linear, a cubic and a quartic self interaction is taken as a source of the energy-momentum tensor, in the Einstein theory with a cosmological constant. Among all the solutions there are two particularly interesting. On the one hand, the spherically symmetric black holes when the cosmological constant is positive; they are shown to be everywhere regular, namely, there is no singularity neither inside nor outside the event horizon. On the other hand, there are spherically symmetric and topological wormholes that connect two asymptotically (anti) de Sitter regions with a different value for the cosmological constant. The regular black holes and the wormholes are supported by everywhere regular scalar field configurations.

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I. INTRODUCTION AND DISCUSSION

The interplay of scalar fields and the gravitational interaction has a long and rich history. Although it dates backs to the Brans-Dicke article [1], it is fair to say that there was a large increment in the interest with Wheeler's no hair conjecture [2]. It states that it is not possible to endow nor to deform a four-dimensional asymptotically flat black hole, regular on and outside the horizon with a scalar field that is regular in the above mentioned region; the domain of outer communications plus its boundary to be precise. This conjecture has been proved in a number of cases (for references and a list of cases where the no hair conjecture is a theorem see [3]) and for real scalar fields in four dimensions there is not much doubt that it is true; a nice account of this history can be found in [4].

As time went by, theoretical and observational arguments moved the community to take the inclusion of a cosmological constant seriously. Allowing the spacetime to be asymptotically of constant curvature changed the picture and a number of black holes were found [5–14]. The no hair conjecture, in these cases, was therefore recast as a no primary hair[1] conjecture for black holes of spherical topology and a scalar field potential derivable from a superpotential [15]. It is worth mentioning that nowadays there is a renewed interest in the no hair conjecture; high precision astronomical observation of the supermassive black holes has been argued to be a way to experimentally test it, see for instance [16].

Scalar fields are also present in the standard cosmological model [17], in any compactification and therefore in most of the extended supergravity theories. As has been previously mentioned, the existence of black holes with scalar fields in anti de Sitter spacetime is, now, widely known. The fact that these hairs are of secondary kind, namely, that there is no integration constant associated to the scalar field, implies that it is not possible to continuously connect the hairy configuration with mass M and a configuration with the same mass and no scalar field. This gives rise to a phase transition at constant free energy in the gravity theory [7]. This kind of second order phase transition has become well known as the gravity dual of a superconductor, for references and a review see [18].

Here a family of wormholes that are asymptotically of constant curvature is constructed in this paper. Anti de Sitter wormholes have gained attention within the holographic context and they have been thoroughly analyzed when the energy momentum tensor vanishes [19]. Since the existence of these kind of objects was disproved when the boundary is globally within the conformal equivalence class of $R \times S^N$, later it was studied whether wormholes with an hyperbolic horizon would be of interest for the AdS/CFT conjecture [20]. Holography was analyzed in a five dimensional wormhole spacetime in [21]. In a more general context these spacetimes have been analyzed within higher dimensional Chern-Simmons theory [22], Horava gravity [23], conformal gravity [24], Lovelock gravity [25] and nonminimally coupled electrodynamics [26,27], as well as evolving Lorentzian wormholes [28], just to mention a few. To add an example in a more simple theory, a family of asymptotically anti de Sitter hairy wormhole solutions is constructed in this paper within

[1]The scalar hair is called primary when there is an extra integration constant associated to it, otherwise it is called secondary.

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the four dimensional Einstein theory when the boundary is $R \times S^2$ or $R \times H^2$.

The wormholes also exist when the cosmological constant is positive. Each of the asymptotic regions have a different value of the cosmological constant and also a different value for the scalar field, corresponding to a different extreme of the potential. In the de Sitter case it is also possible to eliminate the cosmological horizon and the solution becomes an inhomogeneous, anisotropic, cosmology. It starts from a completely homogenous state, with positive constant curvature $\frac{\lambda}{\xi^2}$, evolves to an inhomogeneous state and ends again in a completely homogenous de Sitter space but with a different value for the cosmological constant λ, ξ being an arbitrary parameter of the scalar field potential. It follows that the solution can interpolate between an arbitrarily large cosmological constant in the past and a very small one in the future (or vice versa).

It is also interesting to remark that the black holes have no inner singularity, an issue rather studied when the spacetime is asymptotically flat, and where no example for scalar fields with positive kinetic energy is known, although there are explicit examples when a nonlinear electrodynamic theory is included [29]. (For references and a deeper discussion on regular black holes see the previous reference; for some numerical results in the same directions see [30].) In this paper the inner singularity of the black holes is replaced by another asymptotic region, with a different value for the cosmological and the gravitational constant.

The study of the backreaction of scalar fields on the spacetime also brings in an interesting perspective regarding the dark matter issue. Originally the dark matter problem was related to the impossibility of fitting the orbital velocities of galaxies in clusters with the Newtonian potential, $\Phi = -\frac{GM}{r}$, M being the visible matter within the cluster. The simplest explanation would therefore be that M has another source for which nowadays many candidates exist [31]. When dark matter was proposed the cosmological constant was considered a mathematical curiosity and therefore, at the level of the Einstein equations, the requirement of asymptotic flatness made the assumption on the form of the gravitational potential rather reasonable. However, the inclusion of the cosmological constant allows a larger variety of potentials that can be dominant between the relevant scale of the Newtonian potential and the relevant scale of the cosmological constant. Namely, all the functions that grow slower than r^2 in the asymptotic region and grow slower than r^{-1} close to the surface of the star or black hole. Actually, an upshot of the theoretical studies associated to the detailed analysis of the asymptotic behavior of scalar fields in asymptotically anti de Sitter spacetimes, has been the form of the explicit contribution to the total energy of the spacetime coming from the slow fall off of the scalar field and its backreaction on the metric [32].

In our Universe, better modeled by an asymptotically de Sitter spacetime, the metric can very well fit the observed galaxy rotation curves due to the modification of the gravitational potential. Indeed, since for spherically symmetric solutions in the Schwarzschild gauge, the gravitational potential is related to the metric as $g_{00} = -1 - 2\Phi$. The presence of a scalar field, or any other particle with slow fall off, would give rise to a new kind of potential and therefore the amount of dark matter present in a given model should be reconsidered. Moreover, if dark matter can be modeled as a field and it fills the spacetime from the center of the galaxy all the way to the dark matter halo a simple candidate to analyze this situation is a scalar field. It has already been pointed out that a gravitational potential, arising from the Weyl squared conformal gravity, with the form

$$\Phi = \omega - 2\beta r^{-1} + \gamma r - kr^2 \qquad (1)$$

can give account of the galaxy rotation curves [33]. What is particularly appealing of describing the amount of dark matter using the potential (1) it is that it has three integration constants. Therefore it is possible to assign one to the actual mass of the spacetime, another to the dark matter component and another to the dark energy. This would give a very simple explanation to the fact that the amount of dark matter in every galaxy is different[2] (ranging from 1% to 99%). In this paper a three parametric gravitational potential is shown to arise when the backreaction of a conformally coupled scalar field with a polynomial potential is considered. Instead of having three integration constants it only has one, the usual mass, plus two parameters of the action principle, namely, the cosmological constant and a parameter coming from the scalar potential. We believe that a new integration constant in spherically symmetric configurations would necessarily imply a sort of primary hair which could come either from the matter action or having a pure gravitational origin as in [34].

The outline of the paper is as follows: in the first section the model is presented, the solution is explicitly written and some important features of it remarked. As usual, due to the nonlinearity structure of the field equations the solution has been constructed by an educated guess, therefore no reference to its derivation is made. In the second section, the geometrical characterization of the solutions is done. In the third section, further remarks and comments are done. The notation follows [35]. The conventions of curvature tensors are such that a sphere in an orthonormal frame has positive Riemann tensor and scalar curvature.

[2]Since the mass is an integration constant, within the general relativity realm, there is no fundamental explanation for its value in any gravitating object. If the dark matter content of a gravitating system would also be related to an integration constant no fundamental explanation would be necessary to give account of the difference in the dark matter content of every galaxy.

The metric signature is taken to be $(-,+,+,+)$, $(\partial\phi)^2 = g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi$, $\delta^{\mu\nu}_{\lambda\rho} = \delta^\mu_\lambda\delta^\nu_\rho - \delta^\mu_\rho\delta^\nu_\lambda$. Greek letters are in the coordinate tangent space and Latin letters in the noncoordinate tangent space, $8\pi G = \kappa$ and the units are such that $c = 1 = \hbar$.

II. THE MODEL AND THE SOLUTIONS

As is well known, the Green function of a massless scalar field have support on the light cone on Minkowski spacetime. To extend this behavior to a curved background, such that the Green function goes to its flat space form when the curvature goes to zero, the massless scalar field must be conformally coupled to the scalar curvature [36] (for an interesting generalization of the conformally coupled scalar field to higher dimensions see [37]). Therefore if the conformal coupling is taken as the guide to extend the propagation of a scalar field to an arbitrary background, it would be natural to consider its backreaction in the gravitational field. Indeed, this scalar field gives place to the Bocharova-Bronikov-Melnikov-Bekenstein black hole (for references see [4]). When the cosmological constant is included in the gravity sector and a quartic self interaction is included for the scalar field the Martinez-Troncoso-Zanelli (**MTZ**) black hole arises [6]. Furthermore, the most general Petrov type D solution of General Relativity in vacuum, the Plebanski-Demianski family,[3] has been shown to be an exact solution of this system [11] (it reduces in the nonrotating case to the C-metric like configuration also reported in [12]). Since any way the Weyl invariance of the action is spoiled due to the inclusion of the Einstein-Hilbert term, it is interesting to explore what happens when the scalar field potential is deformed to include non-Weyl invariant terms. In this paper the inspiration have been taken from a realistic real scalar field, the π^0, and its linear sigma model description, the typical soft symmetry breaking linear term have been included[4] as well as a cubic and a quartic self-interaction [38].

The action principle is thus:

$$S(g,\phi) = \int d^4x\sqrt{-g}\left[\frac{R-2\Lambda}{2\kappa} - \frac{1}{2}(\partial\phi)^2 - \frac{1}{12}\phi^2 R - V(\phi)\right], \tag{2}$$

with field equations:

$$G_{\mu\nu} + \Lambda g_{\mu\nu} = \kappa T_{\mu\nu}, \tag{3}$$

$$T_{\mu\nu} = \partial_\mu\phi\partial_\nu\phi - \frac{1}{2}g_{\mu\nu}(\partial\phi)^2 - g_{\mu\nu}V(\phi) + \frac{1}{6}(g_{\mu\nu}\Box - \nabla_\mu\nabla_\nu + G_{\mu\nu})\phi^2 \tag{4}$$

$$\Box\phi = \frac{1}{6}R\phi + \frac{\partial V}{\partial\phi}, \tag{5}$$

where $G_{\mu\nu}$ is the Einstein tensor and $V(\phi) = \alpha_1\phi + \alpha_3\phi^3 + \alpha_4\phi^4$. Exact, analytical, solutions exist in the previous system when $\alpha_4 = -\frac{\kappa\Lambda}{36}$, $\alpha_1 = -\frac{6}{\kappa}\alpha_3$. Using the convenient parametrization $\alpha_3 = -\frac{\Lambda\sqrt{6\kappa}}{9}\frac{\xi}{\xi^2+1}$ the solutions take the following form:

$$ds^2 = \frac{(r+(\xi-1)M)^2}{(r-M)^2}\left[-\left(k\left(1-\frac{M}{r}\right)^2 - \frac{\lambda r^2}{3}\right)dt^2 + \frac{dr^2}{(k(1-\frac{M}{r})^2 - \frac{\lambda r^2}{3})} + r^2 d\Sigma_k\right] \tag{6}$$

$$\phi = \left(\frac{6}{\kappa}\right)^{1/2}\frac{-\xi r + M(\xi-1)}{r+M(\xi-1)}, \qquad \lambda = \frac{\Lambda(\xi^2-1)^2}{\xi^2+1}, \tag{7}$$

where $d\Sigma_k$ is the line element of a surface of constant curvature $k = \pm 1$.

The solution (6) and (7) is real if and only if $|\frac{18\alpha_3}{\Lambda\sqrt{6\kappa}}| < 1$. Another solution is generated by the symmetry of the action

$$\phi \to -\phi, \qquad \xi \to -\xi. \tag{8}$$

In what follows the solutions generated by (8) will be called the negative branch.

Some generic features of these solutions are (straightforward modifications extend all these conclusions to the negative branch):

(i) There are curvature singularities at $r = 0$ and $r = (1-\xi)M$. This last equation also defines the surface where the scalar field is singular.

(ii) The MTZ [6] family of solutions is recovered when $\xi = 0$.

(iii) If $\xi = -1$ then $\lambda = 0$ and the metric is asymptotically flat. There is a naked singularity at $r = 2M$ and at $r = 0$. In this case the negative branch has constant scalar field.

(iv) The metric (6) has two asymptotic regions, $r = \infty$ and $r = M$. The spacetime has a different constant curvature in each of these boundaries as can be seen from the Riemann tensor $\lim_{r\to\infty} R^{\mu\nu}_{\cdot\cdot\lambda\rho} = \frac{\lambda}{3}\delta^{\mu\nu}_{\lambda\rho}$, $\lim_{r\to M} R^{\mu\nu}_{\cdot\cdot\lambda\rho} = \frac{\lambda}{3\xi^2}\delta^{\mu\nu}_{\lambda\rho}$.

(v) It is possible to change ξ by ξ^{-1} with a diffeomorphism. Note that (6) and (7) seems to be two solutions, namely, each one of the real roots of the equation

[3]With event horizons and no conical singularities.

[4]The possibility of including this term was pointed out to us by Alfonso Zerwekh.

$$\alpha_3 = -\frac{\Lambda\sqrt{6\kappa}}{9}\frac{\xi}{\xi^2+1}, \tag{9}$$

that is ξ and ξ^{-1}. However, the map $\xi \longrightarrow \xi^{-1}$ on the configuration (6) and (7) plus the diffeomorphism $r = \rho\xi$, $t = \xi T$ and the reparametrization $M \longrightarrow \xi M$ it is equivalent to the apply the diffeomorphism

$$r = \frac{\rho M}{\rho - M} \tag{10}$$

to the same configuration. Actually, this diffeomorphism interchange the location of infinity, the boundary at $r = \infty$ is mapped to $\rho = M$ and the one at $r = M$ is mapped to $\rho = \infty$.

(vi) The denominator of the scalar field never vanishes for $\xi > 0$ and $r \geq M$.

(vii) There is also a region between $0 < r < M$ which make sense as a black hole when the cosmological constant is negative and the horizon is locally isomorphic to H^2.

(viii) The effective potential $W(\phi) = V(\phi) + \frac{1}{12}\phi^2 R + \kappa^{-1}\Lambda$ of the scalar field is different in each of the boundaries, the evaluation of its first derivative in the configuration (6) and (7) gives:

$$\frac{dW}{d\phi} = \sqrt{\frac{6}{\kappa}}\frac{2\Lambda M r(\xi^2-1)^3(r-M)(r-2M)}{3(\xi^2+1)(r+M(\xi-1))^4} \tag{11}$$

It follows that there is an extremum of the potential at each of the asymptotic regions. The mass of the scalar field at the critical points is

$$\lim_{r\to\infty}\frac{d^2W}{d\phi^2} = -\frac{2\Lambda(\xi^2-1)(2\xi^2+1)}{3(\xi^2+1)},$$
$$\lim_{r\to M}\frac{d^2W}{d\phi^2} = \frac{2\Lambda(\xi^2-1)(\xi^2+2)}{3\xi^2(\xi^2+1)}, \tag{12}$$

$$\lim_{r\to 2M}\frac{d^2W}{d\phi^2} = -\frac{4\Lambda(\xi-1)^2}{3(\xi^2+1)}. \tag{13}$$

(ix) The scalar field acquires a different non trivial vacuum expectation value at each of the boundaries. For the branch (6) and (7) $\phi(\infty) = -(\frac{6}{\kappa})^{1/2}\xi$, $\phi(M) = -(\frac{6}{\kappa})^{1/2}\frac{1}{\xi}$. Therefore it is possible to make a field redefinition to have a scalar field that goes to zero in one of the boundaries but not on both at the same time. It turns out that the potential written in terms of the field $\psi = \phi - \phi(\infty)$ have no linear term at infinity as can be seen from the fact that $\psi = 0$ is a critical point of it. The same happens at the other boundary using the field redefinition $\psi = \phi - \phi(M)$.

(x) The effective gravitational coupling of this model is dynamical, as can be seen by direct inspection of the action principle (2). It has different values at each boundary of the spacetime:

$$\lim_{r\to\infty} G_{\mathrm{eff}} = G(1-\xi^2),$$
$$\lim_{r\to M} G_{\mathrm{eff}} = G(\xi^2-1)\xi^{-2}. \tag{14}$$

However, in a Cavendish experiment the effective coupling is different and it is given by [39]

$$G^*_{\mathrm{eff}} = 2G\frac{1-\frac{\kappa}{9}\phi^2}{(1-\frac{\kappa}{6}\phi^2)(1-\frac{\kappa}{12}\phi^2)}. \tag{15}$$

At each boundary the following values are taken

$$\lim_{r\to\infty} G^*_{\mathrm{eff}} = 2G\frac{2(2\xi^2-3)}{3(2-\xi^2)(\xi^2-1)},$$
$$\lim_{r\to M} G^*_{\mathrm{eff}} = 2G\frac{2\xi^2(3\xi^2-2)}{3(2\xi^2-1)(\xi^2-1)}. \tag{16}$$

It follows that G^*_{eff} is positive at the boundaries if $\frac{1}{2} < \xi^2 < \frac{2}{3}$.

(xi) The stress energy tensor of the scalar field is that of an anisotropic fluid. In an orthonormal frame it has the form of $T^{ab} = \mathrm{diag}(\rho, p_1, p_2, p_2)$. The explicit expression for each of the components is rather involved, however some conclusions can be drawn from

$$\rho + p_1 = -\frac{4M\xi(r-M)}{(r+M(\xi-1))^4\kappa}\left(k\left(1-\frac{M}{r}\right)^2 - \frac{\lambda r^2}{3}\right), \tag{17}$$

$$\rho + p_2 = -\frac{2Mk(r-M)^3(-\xi r + M(\xi-1))}{\kappa r^4(r+M(\xi-1))^3}. \tag{18}$$

From (17) and (18) it follows that for the black holes neither the weak nor the null energy conditions can be everywhere satisfied due to the change of sign of the expression

$$k\left(1-\frac{M}{r}\right)^2 - \frac{\lambda r^2}{3} \tag{19}$$

in the horizons. For anti de Sitter wormholes (19) it is always positive and therefore the above mentioned energy conditions do not hold in the region $r > M > 0$ when $\xi > 0$. For de Sitter cosmologies (19) it is always negative thus $\rho + p_1 \geq 0$ and, moreover, $\rho + p_2 \geq 0$ in the region $r > M > 0$ with $\xi > 0$. Therefore the null energy condition holds in the case of the bouncing cosmologies to be discussed in the next section.

III. GEOMETRICAL CHARACTERIZATION OF THE SOLUTIONS

A. $\Lambda > 0$ and $k = 1$

In this case it is convenient to set $\lambda = \frac{3}{L^2}$. Expression (19) has three positive roots for $\frac{L}{4} > M > 0$:

$$\begin{aligned} r_{++} &= \frac{L}{2}\Big(1 + \sqrt{1 - 4ML^{-1}}\Big), \\ r_{+} &= \frac{L}{2}\Big(1 - \sqrt{1 - 4ML^{-1}}\Big), \\ r_{-} &= \frac{L}{2}\Big(-1 + \sqrt{1 + 4ML^{-1}}\Big). \end{aligned} \tag{20}$$

they satisfy $L > r_{++} > \frac{L}{2} > r_{+} > M > r_{-} > 0$. It follows that when $\infty > r > M$ there is a black hole with the conformal structure of the Kottler solution (also known as the Schwarzschild-de Sitter solution) with the inner singularity replaced by a new asymptotic region. When $M > r > 0$ there is only the cosmological horizon given by r_{-} and therefore the singularity is naked. When $M < 0$ then $\infty > r > 0$ and there is only a cosmological horizon at r_{++}. When $M > \frac{L}{4}$ then r_{++} and r_{+} becomes imaginary and the region between $\infty > r > M$ becomes an inhomogenous bouncing cosmology (for a review see [40]) with the initial condition at $r = M$ being a completely homogenous and isotropic universe with cosmological constant $\frac{\lambda}{\xi^2}$, which after going through an inhomogeneous phase it ends in a completely homogenous de Sitter universe with cosmological constant λ. The bounce is located at

$$r = (1 + \sqrt{\xi})M. \tag{21}$$

It should be noted that the exact solution presented here allows the Universe to evolve from a very large cosmological constant (when $\xi \approx 0$) to a very small one. At the same time the scalar field evolves from a local maximum of the potential to a local minimum. Note also that ξ measures the deviation from conformality of the matter Lagrangian, so a small deviation from conformality is required for this phenomenon to exist.

B. $\Lambda > 0$ and $k = -1$

In this case the metric is everywhere regular, g_{00} never vanishes, and the spacetime can be interpreted as inhomogenous bouncing cosmologies with the time given by the coordinate $r \in [M, \infty]$. The bounce is located at $r = (1 + \sqrt{\xi})M$.

C. $\Lambda < 0$ and $k = -1$

In this case it is convenient to set $\lambda = \frac{3}{L^2}$. Expression (19) has three positive roots for $\frac{L}{4} > M > 0$

$$\begin{aligned} r_{++} &= \frac{L}{2}\Big(1 + \sqrt{1 - 4ML^{-1}}\Big), \\ r_{+} &= \frac{L}{2}\Big(1 - \sqrt{1 - 4ML^{-1}}\Big), \\ r_{-} &= \frac{L}{2}\Big(-1 + \sqrt{1 + 4ML^{-1}}\Big). \end{aligned} \tag{22}$$

In this case there is no cosmological horizon and the two asymptotic regions are separated by two event horizons. All of what it was said in the case of $\Lambda > 0$ and $k = 1$ apply in this case changing the cosmological horizon for a black hole horizon. When $M > r > 0$ there is a black hole with a single event horizon and singularity at $r = 0$. When $M < 0$ then $\infty > r > 0$ and there is also a black hole with a single event horizon at r_{++}. When $M > \frac{L}{4}$ there is a wormhole with the throat located at $r = (1 + \sqrt{\xi})M$. The wormhole interpolates between two asymptotically locally anti de Sitter regions with a different value for the cosmological constant (λ and $\frac{\lambda}{\xi^2}$). The scalar field is everywhere regular whenever $M > 0$ and $\xi > 0$.

D. $\Lambda < 0$ and $k = 1$

This case represent wormhole solutions with the boundary in the conformal class of either $R \times S^2$. The throat is located at $r = (1 + \sqrt{\xi})M$ and the metric interpolates between two asymptotically anti de Sitter regions with a different value for the cosmological constant (λ and $\frac{\lambda}{\xi^2}$). The configuration is everywhere regular whenever $M > 0$ and $\xi > 0$.

IV. FINAL REMARKS

A new class of exact solutions has been presented in this paper. Some of them have the interesting property of having no singularity at all, neither in the spacetime manifold nor in the matter configuration. The singularity inside the black hole is replaced by another asymptotic region, implying, when the horizons are removed, the existence of a new set of either wormholes or bouncing cosmologies. These are the first, exact, asymptotically anti de Sitter wormholes, bouncing de Sitter cosmologies and regular black holes in four dimensions for the Einstein-conformally coupled scalar field system.

The model discussed here (2), indeed, looks rather particular. In principle, it would be desirable to have a classification of all the possible potentials that are compatible with the Einstein equations. Indeed, since the Einstein equations are nonlinear one could expect to find that within certain Petrov class of metrics some restrictions on the scalar potential should hold. This is indeed the case as we will be reported in a forthcoming article.

Finally, to name some open questions let by this work we would like to mention its extension to include the Maxwell field. Another follow-up is to study either thermodynamical properties or stability of the solutions as was done for the MTZ black hole in [41,42], respectively.

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