

Black holes with primary hair in gauged $N = 8$ supergravity

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ABSTRACT: In this paper, we analyze the static solutions for the $U(1)^4$ consistent truncation of the maximally supersymmetric gauged supergravity in four dimensions. Using a new parametrization of the known solutions it is shown that for fixed charges there exist three possible black hole configurations according to the pattern of symmetry breaking of the (scalars sector of the) Lagrangian. Namely a black hole without scalar fields, a black hole with a primary hair and a black hole with a secondary hair respectively. This is the first, exact, example of a black hole with a primary scalar hair, where both the black hole and the scalar fields are regular on and outside the horizon. The configurations with secondary and primary hair can be interpreted as a spontaneous symmetry breaking of discrete permutation and reflection symmetries of the action. It is shown that there exist a triple point in the thermodynamic phase space where the three solution coexist. The corresponding phase transitions are discussed and the free energies are written explicitly as function of the thermodynamic coordinates in the uncharged case. In the charged case the free energies of the primary hair and the hairless black hole are also given as functions of the thermodynamic coordinates.

KEYWORDS: Black Holes, Supergravity Models

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1 Introduction and summary

Just a few years after coining the term “Black Hole” J. A. Wheeler, motivated by the results on the uniqueness of static [1, 2] and stationary black holes [3, 4], proposed the no hair conjecture [5]. While the black hole uniqueness theorems states that given the mass, the electric charge and the angular momentum of an asymptotically flat, four dimensional configuration, which is regular outside and on the event horizon, it is possible to single out the Kerr metric in the vacuum case or the Kerr-Newman in the electrovac case (for references and a nice historical account see the review by Robinson [6]). A comprehensive introduction to the subject can be found in the book by Heusler [7]), the no-hair theorem states that it is not possible to endow neither to deform these space-times, with regular configurations of matter and therefore they would be the unique final state of gravitational collapse observed in nature; certainly, a remarkable conclusion. The conjecture was later extended to the form of theorems in a number of cases, being disproved in the Einstein-Yang-Mills case and Einstein-Skyrme theory (for a modern discussion and references see [8, 9]) and revitalized by the Bocharova-Bronnikov-Melnikov-Bekenstein black hole for a conformally coupled scalar field [10–13]. The four dimensional asymptotically flat case keep attracting the attention of the community on both their theoretical [14, 15] and observational aspects, see for instance [16, 17].

The picture was drastically changed with the inclusion of either higher dimensions or a cosmological constant. Indeed, the very existence of the black ring [18] shows that black holes are not completely characterized by boundary conditions plus conserved charges in dimensions higher than four, and the situation is even more striking in the case of the black Saturn [19] since the condition of vanishing net angular momentum makes the five-dimensional Schwarzschild black hole non-unique. Actually, in five dimensions, it is known to exist an infinite non-uniqueness of black rings coupled to a two form potential. Since the total dipole charge vanishes, the solution it is labeled by a continuous parameter not related with any conserved charge and it can be regarded as a class of primary hair [20].

On the other hand, the Maldacena conjecture turned the attention to the case when the space-time is asymptotically of negative constant curvature and a number of new hairy black holes were found [21–29]. The existence of this hairy black holes, in four dimensions, allowed the discovery of a phase transition when the boundary is in the conformal equivalence class of $\mathbb{R} \times H^2$ [30] in one of the Duff-Liu black holes (which are nicely described in section 6.6 of [31]). When the conformal equivalence class of the boundary is \mathbb{R}^3 and a $U(1)$ vector is included in the system, the second order phase transition can be interpreted as the holographic dual of superconductivity [35, 36].

However, all the black holes that have been constructed until now, in four dimensions, do not defy the spirit of the original no-hair theorems. The value of the scalar hair is actually proportional to the conserved charges of the system and therefore the scalar field does not have its own integration constant. This means that there is no continuous degeneration in the configuration, namely the scalar hair it is switched on or it is not. This kind of hair have been baptized under the name of *secondary hair*. To the best of our knowledge, in four dimensional supergravity there are no known exact¹ regular configurations with an independent parameter associated to the scalar field, such that its value can be arbitrarily varied keeping the conserved charges fixed, namely *primary hair*. The main objective of this paper it is to show that such configurations exist. In particular, in the $U(1)^4$ consistent truncation of the gauged $\mathcal{N} = 8$ supergravity (for a review and references to the long and interesting history of the gauged $\mathcal{N} = 8$ supergravity theory see [31]) one can construct a solution with primary hair.

The key to the construction of a solution with primary hair is the analysis of the discrete symmetries of the Lagrangian and the possible pattern of breaking of such symmetries. The Lagrangian of the $U(1)^4$ consistent truncation of the gauged $\mathcal{N} = 8$ supergravity possesses two discrete symmetries in the scalar fields sector: namely a permutation symmetry of the scalar fields and a reflection symmetry. The hairless case can be considered as a classical solution in which both symmetries are unbroken while the secondary hair corresponds to a spontaneous breaking of the reflection symmetry whereas the permutation symmetry remains unbroken. Thus, one may wonder whether there is a solution which breaks the permutation symmetry as well. Indeed, as it will be shown in the following, such a solution is precisely the sought primary hair.

The paper is organized as follows. In the first section the Duff-Liu static solutions of the gauged $\mathcal{N} = 8$ supergravity are given with a new parametrization for the conserved quantities. The Ashtekar-Das-Magnon (AMD) mass [37, 38] of these configurations in this parametrization is computed. With the mass at hand it is obvious from the parametrization previously introduced that there is a degeneration in the phase space of the theory. Namely, there is more than one black hole for each value of the conserved charges. In particular, it is possible to see that there are three possible uncharged configurations. While two of the uncharged ones are well known, the third is new and has the peculiarity of having vanishing mass and non-vanishing scalar field. By comparing the curvature scalars of this

¹There is numerical evidence of their existence in the case where the topology of the horizon is an sphere [32]. We found that the spherically symmetric, uncharged, hairy black holes are singular. Other non-minimal couplings to non-linear electrodynamics have also been numerically studied [33, 34].

hairy black hole we conclude that there exists a continuous parameter characterizing these configurations, which can be varied without affecting the value of the conserved charges; thus, defining a primary hair. The primary hair is the integration constant that remains when the configuration has vanishing mass, this situation extends to the charged case.

In the second section, the thermodynamic properties of the three uncharged black holes are discussed. The free energy of the three configurations at fixed temperature is compared and it is shown that there exist a critical temperature where all configurations coexist. In the third section the previous results are extended to the charged case.

The notation follows [39]. The conventions of curvature tensors are $[\nabla_\rho, \nabla_\sigma]V^\mu = R^\mu{}_{\nu\rho\sigma}V^\nu$ and $R_{\mu\nu} = R^\rho{}_{\mu\rho\nu}$. The metric signature is taken to be $(-, +, +, +)$, Greek letters are space-time indices and we set $c = 1 = 16\pi G$.

2 The configuration and its charges

The theory considered here is the consistent truncation of $\mathcal{N} = 8$ gauged supergravity [26] described by the action:

$$S = \int d^4x \sqrt{-g} \left(R + \sum_{m=1}^3 \left[-\frac{1}{2} (\partial\varphi_m)^2 + \frac{2}{l^2} \cosh(\varphi_m) \right] - \frac{1}{4} \sum_{i=1}^4 e^{\vec{a}_i \cdot \vec{\varphi}} \left(F^{(i)} \right)^2 \right), \quad (2.1)$$

$$\begin{aligned} \vec{\varphi} &= (\varphi_1, \varphi_2, \varphi_3), & \vec{a}_1 &= (1, 1, 1), & \vec{a}_2 &= (1, -1, -1), \\ \vec{a}_3 &= (-1, 1, -1), & \vec{a}_4 &= (-1, -1, 1). \end{aligned} \quad (2.2)$$

In the sector in which the gauge fields vanish ($F^{(i)} = 0$ for $i = 1, \dots, 4$), this action possesses two discrete symmetries $\varphi_m \rightarrow \varphi_{\pi(m)}$ (where $\pi(m)$ is an arbitrary permutation of $(1, 2, 3)$) and $\varphi_m \rightarrow -\varphi_m$ (reflection). The general static black hole solution can be written by introducing new parameters Q_i ($i = 1, \dots, 4$), such that:

$$\begin{aligned} ds^2 &= -\frac{f}{\sqrt{H}} dt^2 + \sqrt{H} \left(\frac{dr^2}{f} + r^2 d\sigma_k^2 \right), & f &= k + \frac{r^2}{l^2} H - \frac{\mu}{r}, \\ H &= H_1 H_2 H_3 H_4, & H_i &= 1 + \frac{Q_i \mu}{r}, \end{aligned} \quad (2.3)$$

$$\begin{aligned} \varphi_1 &= \frac{1}{2} \ln \left(\frac{H_1 H_2}{H_3 H_4} \right), & \varphi_2 &= \frac{1}{2} \ln \left(\frac{H_1 H_3}{H_2 H_4} \right), \\ \varphi_3 &= \frac{1}{2} \ln \left(\frac{H_1 H_4}{H_2 H_3} \right), & A^{(i)} &= \pm \frac{\sqrt{Q_i + k Q_i^2} \mu}{r + Q_i \mu} dt. \end{aligned} \quad (2.4)$$

Here $d\sigma_k^2$ is the line element of a compact manifold of constant curvature k normalized to $k = \pm 1, 0$. This new parametrization is related to the one of eq. (6.40) and (6.41) of [31] through $Q_i = \sinh(\beta_i)^2/k$. The AMD mass [37, 38] has been used to compute the conserved charges of a large family of rotating black holes in gauged supergravity in [40]. In the present case the AMD mass and the electric charges of the black holes (2.3)–(2.4)

are:

$$M = \frac{\mu(2 + k(Q_1 + Q_2 + Q_3 + Q_4))}{16\pi} \sigma, \tag{2.5}$$

$$q_i = \frac{1}{16\pi} \lim_{r \rightarrow \infty} \int_{\sigma} e^{\vec{a}_i \cdot \vec{\varphi}} * F^{(i)} = \frac{\sqrt{Q_i + kQ_i^2} \mu \sigma}{16\pi}, \tag{2.6}$$

where σ it is the volume of the base manifold.² Note that when $k = -1$, for each value of the charges (M, q_i) there is more than one regular black hole defined by the constants μ and Q_i . In other words, for a given value of the charges there are different black hole configurations and this leads, as it will be discussed in the next sections, to the appearance of interesting phase transitions.

The clearest way to disclose the *non-uniqueness* mentioned above is to analyze the uncharged case ($q_i = 0, \forall i$). It is easy to see from eqs. (2.5) and (2.6) that if one requires that all the charges are vanishing various possibilities arise:

$$q_i = 0 \quad \forall i \Rightarrow Q_i - Q_i^2 = 0 \Rightarrow Q_i = 0 \quad \text{or} \quad Q_i = 1. \tag{2.7}$$

Thus, it is possible to see that the Duff-Liu family of black holes contains three *different* uncharged black holes:

$$\mathbf{0)} \quad \vec{Q} = 0 \quad \Rightarrow \quad h = -1 + \frac{r^2}{l^2} - \frac{\mu}{r}, \quad \varphi_i = 0; \tag{2.8}$$

$$\mathbf{1)} \quad \vec{Q} = (1, 1, 0, 0) \quad \Rightarrow \quad h = -1 + \frac{r^2}{l^2} \left(1 + \frac{\mu}{r}\right), \quad \varphi_1 = \ln\left(1 + \frac{\mu}{r}\right), \quad \varphi_2 = \varphi_3 = 0; \tag{2.9}$$

$$\mathbf{2)} \quad \vec{Q} = (1, 0, 0, 0) \quad \Rightarrow \quad h = \left(-1 + \frac{r^2}{l^2}\right) \sqrt{1 + \frac{\mu}{r}}, \quad \varphi_i = \frac{1}{2} \ln\left(1 + \frac{\mu}{r}\right); \tag{2.10}$$

where

$$\vec{Q} = (Q_1, Q_2, Q_3, Q_4), \quad h = \frac{f}{\sqrt{H_1 H_2 H_3 H_4}}. \tag{2.11}$$

Any other choice of \vec{Q} , corresponds to a relabeling of the scalars or to the change $\varphi \rightarrow -\varphi$. As it has been anticipated in the introduction, the three solutions in eqs. (2.8), (2.10) and (2.9) correspond precisely to the three possible breakings of the discrete symmetries of the scalars sector of the Lagrangian in eq. (2.1).

The black hole in eq. (2.8) is the usual topological Schwarzschild-AdS space-time with a locally hyperbolic horizon: in this case the solutions does not break neither the permutation nor the reflection symmetries of the scalars sector of the action. On the other hand, the solution in eq. (2.10) is the MTZ³ black hole [21–29] in which a third order phase transition occurs [36]. This solution does not break the permutation symmetry of the scalar sector⁴

²The relation between the volume of the compact negative constant curvature manifold and its genus is given by $\sigma = 8\pi(g - 1)$ where $g \leq 2$

³It is easy to see that the change of coordinates $r = \frac{\rho^2}{\rho + M}$ and redefinition $\mu = 4M$, brings the configuration (2.10) to the same form in which is written in [30]. Thus, the MTZ black hole can be seen as a classical solution of the consistent truncation of $\mathcal{N} = 8$ supergravity.

⁴In this case the three components of the scalar multiplet are equal and so $\varphi_m = \varphi_{\pi(m)}$ for any permutation $\pi(m)$.

but it breaks the reflection symmetry since the components of the scalar multiplet are non-vanishing and so $\varphi_i \neq -\varphi_i$. The integration constant μ which gives the “strength” of the scalar field, is proportional to the conserved charges, and therefore can be interpreted as a secondary hair.

Finally, the black hole (2.9) has vanishing mass. As the integration constant μ is proportional to the mass it can in this case be freely adjusted without modifying the conserved charges of the spacetime and it thus becomes a primary hair. Furthermore, from the value of the scalar curvature:

$$R = -\frac{(3r\mu^3 + 27r^2\mu^2 + \mu^2l^2 + 48r^3\mu + 24r^4)}{2(r + \mu)^2r^2l^2}, \tag{2.12}$$

it is possible to see that the change of coordinate that eliminates the parameter μ from the scalar field, $r = \mu\rho$, does not have the same effect in the scalar curvature. We therefore conclude that it is a genuine dynamical parameter; namely a genuine primary hair. We therefore conclude that this parameter is a primary hair. The mechanism presented here has been firstly suggested in [41], where it has been shown that, in the Lovelock theories of gravity, static configurations with vanishing mass support pure gravitational hairs.

As far as the thermodynamical behavior is concerned, standard arguments from statistical mechanics suggest that the configurations with broken symmetries can prevail at low temperatures while one should expect that, at high enough temperatures, the symmetries should be restored. In the following sections we will show that this is indeed the case.

3 Thermodynamics for the uncharged case

The characterization of the thermodynamics of the configurations discussed in the previous section is very clear in the canonical ensemble. Since the gravitational part of the Lagrangians is the Einstein-Hilbert term, it follows that the black holes satisfy the area law. Requiring the Euclidean continuation of the black holes, obtained after performing the Wick rotation $\tau = it$, to be everywhere regular fixes the temperature as the inverse of the period of the Euclidean time $\tau \in [0, \beta)$. In this approach the free energy is defined through the relation $F = M - TS - \sum_i \Phi^{(i)} q^{(i)}$, where $\Phi^{(i)}$ are the electric potentials for each of the Maxwell fields $A^{(i)}$. Since we are considering the uncharged case ($q^{(i)} = 0, \forall i$), the free energy reduces to $F = M - TS$, which is related to the Euclidean action I by $I = -\beta F$. In what follows the quantities associated to the black hole without scalar field, with primary hair and with secondary hair are denoted with a subscript “0”, “1” and “2” respectively. For each of these configurations, the relevant thermodynamical quantities are:

$$T_0 = \frac{2r_+ + 3\mu}{4\pi r_+^2}, \quad S_0 = \frac{\sigma r_+^2}{4}, \quad F_0 = -\frac{\sigma(2r_+ + \mu)}{16\pi}, \tag{3.1}$$

$$T_1 = \frac{\sqrt{\mu^2 + 4l^2}}{4\pi l^2}, \quad S_1 = \frac{\sigma l^2}{4}, \quad F_1 = -\frac{\sigma T l^2}{4}, \tag{3.2}$$

$$T_2 = \frac{\sqrt{l + \mu}}{2\pi l^{\frac{3}{2}}}, \quad S_2 = \frac{l^2 \sqrt{1 + \frac{\mu}{l}} \sigma}{4}, \quad F_2 = -\frac{(2l + \mu)\sigma}{16\pi}. \tag{3.3}$$

It is worth noting that the parameter μ has a different meaning in each of these black holes. In order to decide which are the configurations favoured thermodynamically one should compare the corresponding free energies. However, the above form in which the free energies are seen as functions of μ and r_+ (which are not suitable thermodynamical coordinates) is not convenient. The physical reason is that when one has to decide which one of two or more configurations prevails from the thermodynamical point of view one has to compare the corresponding free energies at the same temperature and voltage. Therefore, the most convenient procedure is to express the masses and the entropies in terms of the corresponding temperatures in the three cases above. This can be done explicitly in the uncharged case. Thus, the free energies in terms of the temperature read

$$F_0 = -\frac{(2T^2\pi^2l^2 + T\pi l\sqrt{4T^2\pi^2l^2 + 3} + 3)\sigma l(2T\pi l + \sqrt{4T^2\pi^2l^2 + 3})}{108\pi}, \quad (3.4)$$

$$F_1 = -T\frac{\sigma l^2}{4}, \quad (3.5)$$

$$F_2 = -\frac{(l + 4T^2\pi^2l^3)\sigma}{16\pi}. \quad (3.6)$$

According to our convention, the configuration with less free energy is thermodynamically the most favorable. It can also be shown that at the critical temperature

$$T_c = \frac{1}{\pi l}, \quad (3.7)$$

the three free energies coincide and so, around $T = 1/(\pi l)$, the three phases coexist since they are equally probable. Below the critical point the secondary hair (which breaks reflections symmetry but does not break the permutation symmetry) is favored. On the other hand, for temperatures larger than T_c the hairless solution is favored and the symmetry is restored. The transition from the hairless solution to the secondary hair is the one reported in [30] in the special case of zero electric charge. As it has been already mentioned, it was shown in [36] that this transition associated to the breaking of the reflection symmetry must be of third order.

The terminology used here for the classification of phase transitions is the one of Ehrenfest where “*n-th order phase transition*” means that the n-th derivative of the free energy is discontinuous at the critical point. Many interesting examples of transitions of order higher than two have been found such as the Gross-Witten-Wadia third order phase transition [42, 43] in a unitary matrix model related to lattice QCD (a nice analysis of the properties of higher order phase transitions is [44]).

In the present case, it can be seen directly⁵ from eqs. (3.4), (3.5) and (3.6), the free energies are continuous at the critical temperature up to their second derivative, and the third derivative as a discontinuity at T_c . The primary hair, which is associated to a spontaneous breaking of both discrete symmetries of (the scalars sector of) the action, is never favored

⁵The advantage to have expressions for the free energies which only depend on the thermodynamical coordinates such as temperature and chemical potentials (as in eqs. (3.4), (3.5) and (3.6)) is that in order to determine the order of the phase transition one does not need to perform any Taylor expansion. The phase transition can be determined analytically.

except at the critical point where it can coexist with the other two phases. This is a quite interesting results since in all the existing literature on primary hairs it has been always argued that primary hair should be highly unstable and disfavored thermodynamically. Thus, one could expect a finite gap among the free energy of the primary hair and the free energies of the other configurations at all the temperatures. However, we have shown that at $T = T_c$ the three free energies coincide up to the second derivatives: this implies that, close to T_c , also the primary hair can be relevant as far as the thermodynamical behavior is concerned.

4 The charged case

It is interesting to study whether the picture described above survives when the black holes support electric charge. The simplest case is when all the charges are equal: $q_i = q$. This implies that either $Q_i = Q_+$ or $Q_i = Q_-$ with

$$Q_{\pm} = \frac{1}{2} \pm \frac{1}{2} \sqrt{1 - 4 \frac{p^2}{\mu^2}}, \tag{4.1}$$

$$p = \frac{16\pi q}{\sigma}. \tag{4.2}$$

Thus, the following bound on the charge must hold:

$$p \leq \frac{\mu}{2}. \tag{4.3}$$

Note that $Q_+ > Q_- > 0$ and therefore the H_i in eq. (2.3) are positive whenever $\mu r > 0$. It follows that the curvature scalars are regular outside of the horizon. In this case there are also three different configurations (up to changes of signs of the scalar fields) which can be characterized by the corresponding Q -vectors as in the previous section. The topological Reisner-Nostdröm-AdS black hole is characterized by the following Q -vector:

$$\vec{Q}_0 = (Q_-, Q_-, Q_-, Q_-). \tag{4.4}$$

On the other hand, the black holes with primary hair and secondary hairs are characterized by \vec{Q}_1 and \vec{Q}_2 respectively:

$$\vec{Q}_1 = (Q_+, Q_+, Q_-, Q_-), \tag{4.5}$$

$$\vec{Q}_2 = (Q_+, Q_-, Q_-, Q_-). \tag{4.6}$$

As in the previous case we had to express the free energies in terms of the temperature only in order to compare them properly, in the present case one should express the three free energies in terms of the temperature T and the chemical potential Φ (which are the correct thermodynamical variables in this case). This can be done explicitly both for the hairless solution and the primary hair (while the corresponding expression for the secondary hair involves the roots of a sixth order algebraic equation so that it cannot be

written analytically):

$$\begin{aligned}
 F_0 &= -\frac{(32T^2\pi^2l^2 + 4T\pi l\sqrt{64T^2\pi^2l^2 + 48 + 3\Phi^2} + 48 + 3\Phi^2)\sigma l(8T\pi l + \sqrt{64T^2\pi^2l^2 + 48 + 3\Phi^2})}{6912\pi}, \\
 F_1 &= -\frac{Tl\sigma\left(16l^2\pi^3 + 4l\pi\sqrt{16l^2\pi^4 + \left(\frac{\Phi}{T}\right)^2 + \left(\frac{\Phi}{T}\right)^2}\right)}{32\pi\sqrt{16l^2\pi^4 + \left(\frac{\Phi}{T}\right)^2}}, \\
 \Phi_2^\pm &= \frac{p}{x_+ + \mu Q_\pm}, \quad S_2 = \frac{l\sqrt{x_+(x_+ + \mu)}}{4\sigma}, \quad F_2 = -\frac{(\mu + 2x_+)\sigma}{16\pi}, \quad \Phi \equiv \sum_{i=1}^4 \Phi^{(i)},
 \end{aligned}$$

where x_+ is the radial location of the horizon for the secondary hair. For small enough chemical potential, Φ , one can show that the picture is very similar to the one of the previous section. Namely, the hairless case is thermodynamically favorable at temperatures higher than the critical temperature while the secondary hair prevails at low temperatures. The critical temperature is a sort of triple point where the three free energies coincide. On the other hand, it can be seen that if one increases the value of the chemical potential Φ then, correspondingly, the range of temperatures where the secondary hair is thermodynamically favored decreases.

5 Conclusions

In this paper the most general static black hole solution of the consistently truncated $\mathcal{N} = 8$ gauged supergravity has been analyzed. Using a new parametrization of the solution space, we showed that for fixed mass and electric charge, in addition to the usual hairless black hole, there exist both a black hole with a secondary scalar hair and another black hole with a primary hair. To the best of our knowledge, this is the first known example of a scalar primary hair in four dimensional (super) gravity. Remarkably, it turned out that the MTZ secondary hair solution is a special case of these solutions when the electric charge is equal to zero. The solutions with scalar hair are associated to a spontaneous symmetry breaking of the discrete permutation and reflection symmetries (of the scalars sector of) the action. In the uncharged case, the free energies can be expressed as functions of the temperature only and it can be shown analytically that the phase transition between the hairless solutions (which prevails at high temperatures) and the secondary hair solution (which prevails at low temperatures) is of third order. The primary hair is thermodynamically disfavored except at the critical point T_c when the three free energies coincide up to the second derivative. Thus, when T is close to T_c the primary hair can coexist with the other configurations and it could be relevant as far as the thermodynamics is concerned. The charges can also be included in the analysis but they do not change the qualitative picture of the uncharged case, especially again there exist zero mass black holes in which case the integration constant becomes a primary hair.

Finally we would like to recall that it was pointed out, in ungauged supergravity, that the primary hair is not precluded by SUSY and therefore it would be possible to construct singular solutions by a duality transformations of the theory [45, 46], and that the primary

hair should be included in the BPS bound. Since the, in this case regular, uncharged solution with primary hair has also vanishing mass it can be regarded as an exotic case of an “extremal” black hole. It would be therefore interesting to see whether the proposal to include the primary hair in the BPS bound can be achieved in this case, and what it would imply.

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