Exact hairy black brane solutions in 5D anti–de Sitter space and holographic renormalization group flows

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We construct a general class of *exact regular* black hole solutions with toroidal horizon topology in five-dimensional anti–de Sitter gravity with a self-interacting scalar field. With these boundary conditions and due to the nontrivial backreaction of the scalar field, the no-hair theorems can be evaded so that an event horizon can be formed. The scalar field is regular everywhere outside the curvature singularity and it vanishes at the boundary where the potential is finite. We study the properties of these black holes in the context of AdS/CFT duality and comment on the dual operators, which saturate the unitarity bound. We present exact expressions for the beta function and construct a c-function that characterizes the renormalization-group flow.

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I. INTRODUCTION

Motivated by AdS/CFT duality [1], there has been extensive work constructing solutions in anti–de Sitter (AdS) spacetime. The finite-temperature dynamics of holographic field theories can be related to the thermodynamics of black holes on the gravity side. Of particular interest are black hole solutions with scalar hair (see, e.g., Refs. [2–10] for some recent applications). However, many of these solutions (for nontrivial scalar potentials) were only generated numerically[1] and so the physics of these solutions in the context of holography can only be partially investigated.

In this letter, we present a class of *exact*, toroidal black hole solutions in AdS_5 gravity with a self-interacting scalar field—the nontrivial backreaction of the scalar field can produce an event horizon. To the best of our knowledge, this is the first example of a five-dimensional exact, regular, neutral, hairy black hole solution. These solutions are important for understanding the no-hair theorems when the horizon does not have spherical topology (see, e.g., Ref. [11]) and are constructed by using the method of Ref. [12] (see, also, Ref. [13]).

Within AdS/CFT duality, the physics of this family of solutions is very rich. The classical (super)gravity regime corresponds to the large-N, strong 't Hooft coupling limit of the gauge theory. Using exact AdS gravity backgrounds,[2] it is possible to investigate in detail theories that have some similar features with four-dimensional QCD, e.g. confinement/deconfinement transitions [14], nontrivial speeds of sound, bulk viscosities [15], renormalization-group (RG) flows (β functions) [16–18], hydrodynamic properties [19], etc. The existence of exact hairy black branes in AdS opens the exciting possibility of investigating the strong regime of certain field theories in a well-controlled setting.

Interestingly enough, a part of the scalar potential we consider is controlled by a tunable parameter, α, so that we can obtain exact domain wall solutions when $\alpha = 0$ and exact planar black holes when $\alpha \neq 0$. This allows us to explore how the extended literature on domain walls and fake supergravity potentials apply to this case. Since the black hole solutions are analytic, we obtain exact beta functions. We would like to point out that the expressions for the domain wall and black hole beta functions coincide, though the physics is obviously different.

For tachyonic scalars, it was shown in Ref. [20] that the slow a-branch[3] contributes to the boundary charges. The mass was chosen to be $m^2 = -3l^{-2}$, where l is the AdS_5 radius, which is indeed above the Breitenlohner-Freedman bound and it belongs to the mass spectrum of the lowest short multiplet of PSU(2, 2|4). This implies that the linearized spectrum of this model coincides with the one of type IIB supergravity compactified on S^5. Moreover, the mass saturates the unitarity bound; in Refs. [20,21], it was argued that, when the unitarity bound is violated $\Delta_- \leq 1$, the Klein-Gordon inner product is divergent when evaluated for these configurations. Indeed, the Klein-Gordon inner product is well defined only for solutions of the

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[1]In fact, in many relevant examples for holography, by imposing constraints on the form of the scalar potential just the asymptotic and near horizon expansions of the metric are presented.

[2]By including additional background fields, which maintain the asymptotic boundary behaviour, the conformal symmetry can be broken.

[3]The asymptotic form of the scalar field is $\phi = \frac{a}{r^{\Delta_-}} + \frac{b}{r^{\Delta_+}}$, where $\Delta_\pm = 2(1 \pm \sqrt{1 + \frac{l^2 m^2}{4}})$ and l is the AdS radius. The slow branch is the one controlled by Δ_-.

linearized Klein-Gordon field equation, namely perturbations over the configurations that we are considering here. These linearized perturbations seems to be normalizable only if Dirichlet or, as is the case of this work, AdS-invariant boundary conditions are imposed on them [22].

We will present the thermodynamical properties of our solutions and also discuss in detail some of their interesting "holographic" features.

II. THE SETUP

The action we are interested in is

$$I[g_{\mu\nu},\phi]=\frac{1}{2\kappa}\int_M d^5x\sqrt{-g}\left[R-\frac{1}{2}\partial_\mu\phi\partial^\mu\phi-V(\phi)\right]-\frac{1}{\kappa}\int_{\partial M}d^4x\sqrt{-\gamma}K, \tag{1}$$

where $V(\phi)$ is the scalar potential and we use the convention $\kappa=8\pi G_N$. Here, K is the trace of the extrinsic curvature of the boundary ∂M as embedded in M and γ_{ab} is the induced metric on the boundary. Since we set $c=1=\hbar$, $[\kappa]=M_P^{-3}$ where M_P is the five-dimensional Planck scale.

The equations of motion for the metric and dilaton are

$$E_{\mu\nu}=R_{\mu\nu}-\frac{1}{2}g_{\mu\nu}R-\frac{1}{2}T^{\phi}_{\mu\nu}=0,\qquad \frac{1}{\sqrt{-g}}\partial_\mu(\sqrt{-g}g^{\mu\nu}\partial_\nu\phi)-\frac{\partial V}{\partial\phi}=0, \tag{2}$$

where the stress tensor of the matter fields is

$$T^{\phi}_{\mu\nu}=\partial_\mu\phi\partial_\nu\phi-g_{\mu\nu}\left[\frac{1}{2}(\partial\phi)^2+V(\phi)\right]. \tag{3}$$

For simplifying our analysis, we use the following ansatz for the metric:

$$ds^2=\Omega(x)\left[-f(x)dt^2+\frac{\eta^2dx^2}{f(x)}+d\Sigma^2\right], \tag{4}$$

where the parameter η was introduced to obtain a dimensionless radial coordinate x, $\Omega(x)$ is the conformal factor, and $d\Sigma^2=\sum_{a=1}^3 dx_a^2$.

There are three independent (combinations of) equations of motion. Two of them are

$$E_t^t-E_r^r=0\Rightarrow\phi'^2=\frac{9(\Omega')^2-6\Omega\Omega''}{2\Omega^2}, \tag{5}$$

$$E_t^t-E_a^a=0\Rightarrow 2\Omega f''+3\Omega' f'=0, \tag{6}$$

where the derivatives are with respect to x. The remaining equation is more complicated, but it is worth emphasizing that the potential can be obtained by solving it.

III. SOLUTIONS

As in Ref. [12], we choose the conformal factor

$$\Omega(x)=\frac{\nu^2x^{\nu-1}}{\eta^2(x^\nu-1)^2} \tag{7}$$

so that we can solve Eq. (5) to get

$$\phi=l_\nu^{-1}\ln(x),\qquad l_\nu^{-1}=\sqrt{\frac{3(\nu^2-1)}{2}}, \tag{8}$$

where $\nu^2>1$. A canonically normalized scalar field has the dimension $[l_\nu]=[L^{\frac{3}{2}}]$, but with our normalization the scalar field is dimensionless. The parameter ν labels different hairy solutions.

The other metric function is

$$f(x)=-\frac{\Lambda}{6}+\alpha\left[\frac{4}{3(\nu^2-25)(9\nu^2-25)}+\frac{x^{\frac{5}{2}}}{12\nu^3}\left(\frac{x^{\frac{3\nu}{2}}}{3(3\nu+5)}+\frac{x^{-\frac{3\nu}{2}}}{3(3\nu-5)}-\frac{x^{\frac{\nu}{2}}}{(\nu+5)}-\frac{x^{-\frac{\nu}{2}}}{(\nu-5)}\right)\right] \tag{9}$$

and the dilaton potential is

$$V(\phi)=\frac{(9\nu^2-25)e^{-\phi l_\nu}}{4\nu^2}\left[\Lambda-\frac{8\alpha}{(\nu^2-25)(9\nu^2-25)}\right]\left[\frac{(\nu+1)}{2(3\nu-5)}e^{\nu\phi l_\nu}+\frac{(\nu-1)}{2(3\nu+5)}e^{-\nu\phi l_\nu}+\frac{5(\nu^2-1)}{(9\nu^2-25)}\right]$$
$$+\frac{\alpha e^{\frac{3}{2}\phi l_\nu}}{2\nu^3}\left[\frac{5(\nu^2-1)}{\nu^2-25}\left(\frac{e^{\frac{\nu}{2}\phi l_\nu}}{3\nu+5}+\frac{e^{-\frac{\nu}{2}\phi l_\nu}}{3\nu-5}\right)+\frac{1}{3}\left(\frac{(\nu-1)e^{\frac{3\nu}{2}\phi l_\nu}}{(\nu+5)(3\nu+5)}+\frac{(\nu+1)e^{-\frac{3\nu}{2}\phi l_\nu}}{(\nu-5)(3\nu-5)}\right)\right]. \tag{10}$$

Note that the potential and the configuration are invariant under the change of $\nu\to-\nu$. At this point we observe that the potential has two parts, one of which is controlled by the cosmological constant $\Lambda=-\frac{6}{l^2}$ and the other by α which is an arbitrary parameter with the same dimension as Λ, namely $[\alpha]=[L^{-2}]$. The rank of the coordinate x can be taken to be either $x\in(0,1]$ or $x\in[1,\infty)$. The scalar field is negative in the first case, but positive in the other range. Since the dilaton potential has no obvious symmetry, this allows one to cover it completely. The singularities of the metric (and the scalar field) are at $x=0$ and $x=\infty$, but they are enclosed by an event horizon. It is easy to see that, for any value of Λ, ν, and x_+, there is an α such that $f(x_+)=0$. Indeed, this simply follows from the fact that $f(x)$ is linear in α. Therefore, there are black holes in an open set of the parameter space. Moreover, a simple inspection of the function (9) shows that it is regular for every positive $x \neq 0$ and $x \neq \infty$.

At the boundary, $x = 1$, the scalar field vanishes, the potential is $V = 2\Lambda$ and, moreover, $\frac{dV}{d\phi}|_{\phi=0} = 0$, $\frac{d^2V}{d\phi^2}|_{\phi=0} = -\frac{3}{l^2}$. Therefore, the scalar mass is indeed above the Breitenlohner-Freedman bound and it matches the mass of some scalars in the spectrum of the lowest short multiplet of PSU(2, 2|4). A numerical exploration of other extrema seems to indicate that the only one is this maximum at the origin of the potential.

As is well known, the only relevant energy condition in AdS spacetimes is the null energy condition, which is trivially satisfied for hairy solutions of the form (4). In an orthonormal frame e^a_μ, the energy-momentum tensor has the form $T^\phi_{\mu\nu} e^\mu_a e^\nu_b = \mathrm{diag}(\rho, p_1, p_2, p_2, p_2)$ where

$$\rho + p_1 = \frac{3(\nu^2 - 1)f(x)}{4\Omega(x)\eta^2} \geq 0, \qquad \rho + p_2 = 0. \tag{11}$$

IV. THERMODYNAMICS

To gain some intuition, let us first consider the limit for which we can recover the usual no-hair planar black hole. This is possible in the limit $\nu = 1$ when the conformal factor becomes $\Omega(x) = [\eta(x-1)]^{-2}$. To obtain the usual planar coordinates, we should match the factors of $d\Sigma^2$ so that $\Omega(x) = r^2$ and so we get (for the positive branch)

$$x = 1 + \frac{1}{\eta r}, \tag{12}$$

and for $\nu = 1$ we obtain

$$f\Omega = r^2\left[-\frac{\Lambda}{6} + \left(\frac{1}{3}\right)\left(\frac{\alpha}{96\eta^4}\right)\frac{1}{r^4}\right] = \frac{r^2}{l^2} + \frac{1}{3}\left(\frac{\alpha}{96\eta^4}\right)\frac{1}{r^4}. \tag{13}$$

The mass of the planar black hole was computed in Ref. [23] by using counterterms,

$$M = \frac{3}{16\pi G_N} m V_3 = -\left(\frac{1}{16\pi G_N}\right)\frac{\alpha}{96\eta^4} V_3, \tag{14}$$

where m is the mass parameter of the planar black hole and V_3 is the infinite volume of the space with the metric $d\Sigma^2$. It is worth noticing that the event horizon exists only when α is negative, and therefore the mass is always positive (as is also the temperature; see below).

Since in the planar case there is no Casimir energy, it is more convenient when the scalars are turned on to use the method of Ashtekar, Das, and Magnon [24]. Interestingly enough, the mass has the same form as for the planar black hole discussed above,

$$M_{\mathrm{hairy}} = -\left(\frac{1}{16\pi G_N}\right)\frac{\alpha}{96\eta^4} V_3, \tag{15}$$

but now the parameter α plays the role of a multiplicative coupling constant in the Lagrangian and cannot be eliminated as in the previous case, when just the mass parameter characterizes the solution.

To check the first law, we also need the temperature and entropy of the hairy black hole, which are

$$T = \frac{f'(x)}{4\pi\eta}\Bigg|_{x=x_+} = -\frac{\alpha|x_+^\nu - 1|^3}{288\pi\eta\nu^3 x_+^{\frac{3}{2}(\nu-1)}}, \tag{16}$$

$$S = \frac{A}{4G_N} = \frac{\Omega(x_+)^{\frac{3}{2}}}{4G_N} V_3 = \frac{\nu^3 x_+^{\frac{3}{2}(\nu-1)}}{4G_N\eta^3|x_+^\nu - 1|^3} V_3, \tag{17}$$

where x_+ is the location of the horizon, which is the largest root of $f(x_+) = 0$.

Now, it is easy to check that the first law is satisfied if we work with densities of the relevant physical quantities. It is well known that, with the Wick rotation $t \to i\tau$, the Euclidean path integral yields a thermal partition function. Then, the Euclidean black hole has the interpretation of a saddle point in this path integral and so the gravity action evaluated for the classical solution is the leading contribution to the free energy,

$$F = I_E T = M - TS = \frac{\alpha}{3^2 2^9 \pi G_N \eta^4} V_3 < 0. \tag{18}$$

At this point, we would like to also comment on the physics of planar black holes when identifications in the horizon geometry are made so that the horizon becomes a torus.[4] In this case, since there exist compact dimensions, there is a (negative) Casimir contribution to the black hole mass. Since this is constant, it does not play any role for the first law. Also, the quantum statistical relation is satisfied because the action also gets a contribution that will cancel the Casimir contribution from the mass.

However, the phase diagram is drastically changed in this case. As in the no-hair planar black hole case [25], we found [26] that there is a similar phase transition between the hairy black holes and a hairy AdS soliton that is obtained by a double analytic continuation (as in Ref. [27]) from the planar hairy black hole.

V. HOLOGRAPHY

In the AdS/CFT context, the conformal field theory (CFT) on the boundary is the UV fixed point of a four-dimensional QFT. Deformation of the gauge theory by the addition of relevant operators is one way to reduce its symmetries. By using the gravity side of the correspondence (deformations of AdS_5), one can obtain holographic RG flows [16,18] corresponding to nonconformal field theories. Here, we consider the decoupling limit of AdS/CFT duality at finite temperature [1] when the bulk theory is a black hole,

[4]When considering black holes with hyperbolic horizon topology, by doing identifications in the horizon geometry one can obtain topological black holes whose horizon topology is a generalization of the Riemann surfaces.

$$ds^2 = b^2(u)\left[-f(u)dt^2 + d\Sigma^2 + \frac{du^2}{f(u)}\right], \tag{19}$$

and the boundary where the field theory is at the UV fixed point is $u \to 0$.

The running of the gauge coupling is simply a consequence of the dilaton being nonconstant. The beta function of the theory, as a function in terms of the background solution, is

$$\beta(e^\phi) = b(u)\frac{de^{\phi(u)}}{db(u)}. \tag{20}$$

Given this definition, it is straightforward to write the beta function in the coordinates of Eq. (4),

$$\beta(e^\phi) = 2\Omega\frac{de^{\phi(x)}}{d\Omega} = 2\frac{\Omega}{\Omega'}\frac{de^{\phi(x)}}{dx} = 2\frac{\Omega}{\Omega'}\frac{dx^{l_\nu^{-1}}}{dx}$$
$$= -\frac{2}{l_\nu}\frac{e^{\nu\phi l_\nu} - 1}{e^{\nu\phi l_\nu}\nu + e^{\nu\phi l_\nu} + \nu - 1}e^\phi. \tag{21}$$

As expected, we can easily see that the β function vanishes at the conformal UV fixed point.

In general, the beta function is derived from the moduli potential via a superpotential. In our case, since we have exact solutions we can directly use Eq. (20). However, for completeness, let us also discuss the case $\alpha = 0$ for which the superpotential can be explicitly obtained. Unlike the $\alpha \neq 0$ case, for $\alpha = 0$ we obtain a domain wall with a naked singularity. The superpotential is

$$W(\phi) = \frac{\sqrt{6}}{2\nu l}\left[(\nu-1)e^{-\frac{(\nu+1)\phi l_\nu}{2}} + (\nu+1)e^{\frac{(\nu-1)\phi l_\nu}{2}}\right], \tag{22}$$

where

$$V(\phi) = 3\left[\frac{dW(\phi)}{d\phi}\right]^2 - 2W(\phi)^2. \tag{23}$$

We can then compute the β function from the superpotential as in Ref. [17], $\beta(\phi) = -\frac{1}{W(\phi)}\frac{dW(\phi)}{d\phi}$, and see that, indeed, it has the same expression as Eq. (21) (the β function does not depend explicitly on α).[5]

Let us now compute the c-function that is an off-shell generalization of the central charge. The central charge counts the number of massless degrees of freedom in the CFT; in other words it counts the ways in which the energy can be transmitted. The coarse graining of a quantum field theory removes the information about the small scales and so, for a QFT RG flow, there should exist a c-function that is decreasing monotonically from the UV regime (large radii in the dual AdS space) to the IR regime (small radii in the gravity dual) of the QFT.

[5] In fact, since in Ref. [17] the beta function is defined as $\beta = a\frac{d\phi}{da}$ and not with respect to e^ϕ, the results match up to an e^ϕ factor.

Here, we follow closely Ref. [18]. By using the null energy condition (which holds in our case) and the equations of motion, one can show that for an ansatz

$$ds^2 = -a(r)^2dt^2 + \frac{dr^2}{c^2(r)} + b^2(r)d\Sigma^2 \tag{24}$$

the c-function is $C(r) = C_0\frac{a^3}{b'^3c^3}$, which in our coordinates becomes

$$C(x) = 8C_0\frac{\Omega^{\frac{9}{2}}}{\Omega'^3} = 8C_0\frac{\nu^3}{\eta^3}\frac{x^{\frac{3}{2}(\nu+1)}}{(\nu x^\nu + \nu + x^\nu - 1)^3} \tag{25}$$

and has the right properties. The constant is fixed by the entropy of the black hole (in the IR regime). For the planar neutral black hole, the conformal radius is $b(r) = r$ and the c-function is constant: the flow is trivial (this corresponds to the "hairless" limit $\nu = 1$).

We see that the c-function is completely determined by the conformal factor and the other function in the metric, $f(x)$, does not play any role. We can understand why since, in the dual description, the black hole is interpreted as a thermal state. Therefore, the c-function should remain unchanged when we excite a finite-temperature vacuum in the same theory (with the same degrees of freedom).

We would like to explain why the parameter α controls the physics in the deep infrared only. For this, let us expand the potential around the boundary ($\phi = 0$),

$$V(\phi) = -\frac{12}{l^2} + \frac{(\nu^2-1)}{l^2}\left(-\frac{9l_\nu^2}{4}\phi^2 + \frac{l_\nu^3}{4}\phi^3 - \frac{3(\nu^2-2)l_\nu^4}{16}\phi^4 + \frac{(7\nu^2-18)l_\nu^5}{80}\phi^5 + \frac{(420 - 12\nu^2(3\nu^2+5) + 5\alpha l^2)l_\nu^6}{5760}\phi^6 + \cdots\right), \tag{26}$$

where the first nontrivial contribution of the α term was included.

Interestingly, it was shown in Ref. [20] that the $O(\phi^3) + \cdots$ terms in the scalar potential are irrelevant to calculating the Hamiltonian generators. Since for our solutions α appears at the sixth order in the expansion, this part of the scalar field potential does not backreact strongly in the boundary, but in the bulk the self-interaction term controlled by α is responsible for the appearance of the horizon.

We change the coordinates

$$\ln x = \frac{1}{\eta r} - \frac{1}{2\eta^2r^2} - \frac{\nu^2-9}{24\eta^3r^3} + \frac{\nu^2-4}{12\eta^4r^4} + \frac{(9\nu^2-25)(\nu^2-25)}{1920\eta^5r^5} \tag{27}$$

so that our metric matches the asymptotic form of Ref. [20].

Solving the linearized scalar field equations with a constant mass term, there are two independent solutions. Since for our solutions $m^2 l^2 = -3$, we obtain $\Delta_+ = 3$, $\Delta_- = 1$ and so the behavior of this scalar at the boundary is

$$\phi = \frac{a}{r}(1 + \cdots) + \frac{b}{r^3}(1 + \cdots). \tag{28}$$

Depending on the form of the potential, there is a logarithmic term that can appear when Δ_+/Δ_- is an integer [20] (see, also, Ref. [28]); however, it is not present in this solution.

(Super)gravity fields do not scale under four-dimensional conformal transformations, so a must have dimension 1 and b must have dimension 3 which corresponds to a being a source (i.e. a mass) and b being its vacuum expectation value (or condensate). Since $m^2 < 0$, the dual operators are relevant and so the backreaction will be so that the metric remains asymptotically AdS, reflecting the conformal fixed point in the UV. One can check this observation in the coordinates of Ref. [20] for which we obtain the deviations from the AdS metric at infinity as follows. The change of coordinates (27) brings the metric to standard AdS coordinates. Indeed, the metric takes the form

$$ds^2 = -\frac{r^2}{l^2}dt^2 + \frac{l^2}{r^2}dr^2 + r^2\delta_{mn}dx^m dx^n + h_{\mu\nu}dx^\mu dx^\nu, \tag{29}$$

where $h_{\mu\nu}$ are the deviations from AdS. Using Eq. (27) one can then obtain that the falloff of the metric is the one predicted in Ref. [20] for the generic case. We would like to point out that the Ashtekar-Das-Magnon mass is exactly the r^{-2} factor in the lapse function.

VI. DISCUSSION AND FUTURE DIRECTIONS

Finding exact solutions of Einstein field equations with (or without) matter sources is a subject of long standing interest. Indeed, exact solutions with nontrivial moduli potentials can be important, in particular, for phenomenological bottom-up approaches in string theory (see, e.g., Refs. [15,29]). Some predictions of the gauge/gravity correspondence may be universal enough as to apply to QCD, at least in certain regimes.

One important result we have obtained is the existence of a nontrivial RG flow and, for future work, it will be interesting to understand how much the dual field theory "mimics" QCD as in Refs. [15,29]. In particular, one could compute the speed of sound and bulk viscosity and see which values of the "hairy parameter" (ν) are interesting. There is also a conjecture by Buchel on a dynamical bound of the bulk viscosity [30] that can be checked with our solutions.

Let us comment now on the limit $\alpha = 0$ that is equivalent with considering a vanishing mass for our solution. We see that the potential is still nontrivial, but in this case we obtain a naked singularity. Once the self-interaction of the scalar field, which is proportional to the parameter α is considered the singularity gets "dressed" by a horizon and a regular black hole solution is obtained. This is one concrete example for which new terms/corrections of the potential convert singular solutions into regular black holes with finite horizon area.

One possible extension of our work is to include gauge fields. The physics and phase structure are richer in this case and it is possible to investigate the physics at zero temperature due to the existence of extremal black hole solutions. When the scalar field is nonminimally coupled to the gauge fields and the scalar charge is determined by the charges and mass, the hair is referred to as "secondary." The scalar charge is not protected by a gauge symmetry and so it is not a conserved charge. It was shown in Ref. [31] that the scalar charges, even if not conserved, can appear in the first law of thermodynamics (the interpretation of this result, though, should be taken with caution [32]).

In the presence of gauge fields, we were also able to construct exact solutions in AdS [33].[6] In the extremal limit, since the flow of the moduli is interpreted as an RG flow, the attractor mechanism acts as a no-hair theorem [35].

There are other interesting directions, which can be investigated. For example, using the results in Sec. IV, it is straightforward to obtain the phase structure of our solutions. When the foliation of Euclidean AdS is $R_3 \times S^1$ (S^1 is the Euclidean time circle), the only scale in the system (the temperature) can be scaled out via conformal invariance. Thus, the $\mathcal{N} = 4$ theory on R_3 cannot have a phase transition at any nonzero temperature [36]. However, there are first-order transitions between these black holes and the corresponding soliton (which is the solution with the minimum energy within the solutions with the same boundary conditions), which is constructed by a double analytic continuation similar to the one in Ref. [27]. It is important to emphasize that there also are second-order phase transitions similar to the ones discussed in Ref. [37]. These results will be presented in a companion paper [26].

To end our discussion we would like to point out that with our method we can also generate exact solutions with scalar and gauge fields when the cosmological constant is positive. These solutions can be described along the lines of Ref. [38] (e.g., computing the corresponding c-functions).

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[6] The existence of solutions for which both modes of the scalar are normalizable was numerically proven in Ref. [34].

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