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On attractor mechanism of AdS_4 black holes

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ABSTRACT

We construct a general family of exact non-extremal 4-dimensional black holes in AdS gravity with $U(1)$ gauge fields non-minimally coupled to a dilaton and a non-trivial dilaton potential. These black holes can have spherical, toroidal, and hyperbolic horizon topologies. We use the entropy function formalism to obtain the near horizon data in the extremal limit. Due to the non-trivial self-interaction of the scalar field, the zero temperature black holes can have a finite horizon area even if only the electric field is turned on.

1. Introduction

In recent years, it became clear that the Anti-deSitter(AdS)/conformal field theory (CFT) may be also an useful tool for studying strongly-coupled toy models of condensed matter systems (see, e.g., [1] and references therein).

A careful analysis of charged dilaton black holes in AdS_4 was initiated in [2]. The focus of that work was on extremal and near-extremal solutions when the scalar potential is constant (proportional to the cosmological constant). Although a lot of work was done in this direction, it seems that *exact* hairy AdS black hole solutions with a non-trivial dilaton potential are not easy to construct. In the seminal work [3] (see, also, [4]), a concrete method for solving the equations of motion was proposed and families of *asymptotically flat* hairy black hole solutions were constructed. That is, to rewrite the equations of motion as Toda equations and use the well-known techniques to solving them. Unfortunately, this method is not powerful enough to get general exact AdS_4 hairy black holes [5].

In this work we present a general family of *exact* 4-dimensional non-extremal black holes in AdS gravity with $U(1)$ gauge fields non-minimally coupled to a scalar field. Recently, a class of exact AdS_4 black hole solutions was obtained [6,7], though in these works a different method was used (see also [8–12]). Our solutions are more general and are of particular interest to clarifying aspects of the AdS/CMT correspondence because they can provide information about distinct holographic phases of matter [13]. Also, they can be useful in the context of fake supergravity [14]. We consider a non-trivial dilaton potential and obtain the corresponding superpotential in a particular case.

However, there is another important motivation for our work. That is, these solutions can exhibit new features which are absent for theories with a trivial dilaton potential [2]. For example, the extremal limit black hole solutions can have a finite horizon area even if just one electric field is turned on. To the best of our knowledge these are the first *analytic* asymptotically AdS black holes with this property. From this point of view, this is an interesting example where the attractor mechanism [15] is at work.[1] As was observed in [16,20], the moduli flow can be interpreted as an RG flow in the context of AdS/CFT duality. Therefore, different kinds of attractors will characterize the IR behavior of the dual CFT which is at zero temperature but is now deformed by the addition of a chemical potential.

The Letter is organized as follows: in Section 2 we present the set-up. Then, in Section 3, we construct the most general solution for an arbitrary exponential dilaton coupling of the gauge field and present some concrete examples. In Section 4 we discuss the extremal limit in the context of attractor mechanism. We conclude the Letter with comments and present some possible further directions in Section 5.

2. General formalism

In this section we follow closely [21] (see, also, [22]) where the asymptotically flat 'cousins' of our black hole solutions were constructed.

We are interested in a generic action of the form

[1] It is well known that extremal black holes exhibit the attractor mechanism regardless of supersymmetry [16,17] and this is at the basis of computing the entropy of extremal non-BPS black hole [18,19].

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$$I[g_{\mu\nu}, A_\mu, \phi] = \frac{1}{2\kappa}\int d^4x\sqrt{-g}\left[R - \frac{1}{4}e^{\gamma\phi}F^2 - \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - V(\phi)\right] \tag{1}$$

where the gauge coupling and potential are functions of the dilaton and we use the convention $\kappa = 8\pi G_N$. Since we set $c = 1 = \hbar$, $[\kappa] = M_P^{-2}$ where M_P is the reduced Planck mass. The equations of motion for the gauge field, dilaton, and metric are

$$\nabla_\mu\left(e^{\gamma\phi}F^{\mu\nu}\right) = 0$$

$$\frac{1}{\sqrt{-g}}\partial_\mu\left(\sqrt{-g}g^{\mu\nu}\partial_\nu\phi\right) - \frac{\partial V}{\partial\phi} - \frac{1}{4}\gamma e^{\gamma\phi}F^2 = 0$$

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = \frac{1}{2}\left[T^{\phi}_{\mu\nu} + T^{EM}_{\mu\nu}\right]$$

where the stress tensors of the matter fields are

$$T^{\phi}_{\mu\nu} = \partial_\mu\phi\partial_\nu\phi - g_{\mu\nu}\left[\frac{1}{2}(\partial\phi)^2 + V(\phi)\right]$$

$$T^{EM}_{\mu\nu} = e^{\gamma\phi}\left(F_{\mu\alpha}F_\nu^{\cdot\alpha} - \frac{1}{4}g_{\mu\nu}F^2\right)$$

The ansatz for the metric is more general than in asymptotically flat case [21], because in AdS there also are black holes with toroidal and hyperbolic horizon topologies:

$$ds^2 = \Omega(x)\left[-f(x)\,dt^2 + \frac{\eta^2\,dx^2}{f(x)} + d\Sigma_k\right] \tag{2}$$

where the parameter η was introduced to obtain a dimensionless radial coordinate x and $\Omega(x)$ is the conformal factor. $d\Sigma_k$ is a surface of constant curvature normalized to be $k = \pm 1$ or 0 and which can be conveniently parameterized as[2]

$$d\Sigma_k = \frac{dy^2}{1 - ky^2} + \left(1 - ky^2\right)d\varphi^2 \tag{3}$$

This is the most general static asymptotically locally AdS ansatz, characterized by only two unknown functions. The equation of motion for the gauge field and Bianchi identity can be solved by the following ansatz of the field strength:

$$F = Q\,e^{-\gamma\phi}\,dx \wedge dt + P\,dy \wedge d\varphi \tag{4}$$

The strategy for finding exact solutions is similar with the one in the asymptotically flat case [21] and we do not repeat all the steps here. We use the same conformal factor

$$\Omega(x) = \frac{\nu^2 x^{\nu-1}}{\eta^2(x^\nu - 1)^2} \tag{5}$$

In these coordinates, the scalar can be written in a nice form

$$\phi(x) = l_\nu^{-1}\ln(x) + \phi_0 \tag{6}$$

where $l_\nu = (\nu^2 - 1)^{-\frac{1}{2}}$ plays the role of a characteristic length scale of the dilaton. There is a $\pm$ ambiguity in the integration of the dilaton equation, which corresponds to a discrete degeneration in the black hole family. Indeed, from (5) it follows that the conformal factor has a pole of order two at $x = 1$ where the conformal infinity is 'located'. The fact that (5) is regular in the region $x \in (0, 1)$ and $x \in (1, \infty)$ allows to pick any of these intervals as the domain of coordinates, one corresponding to a negative scalar and the other corresponding to a positive one.

[2] The usual canonical form is obtained with the following change of coordinates: $k = 1 \Rightarrow y = \cos\theta$ and $k = -1 \Rightarrow y = \sinh\theta$.

The remaining metric function satisfies the following differential equation:

$$\frac{1}{\Omega}\left(\Omega f'\right)' + 2\eta^2 k - \frac{e^{-\gamma\phi}Q^2 + e^{\gamma\phi}\eta^2 P^2}{\Omega} = 0 \tag{7}$$

which can be exactly integrated.

3. Exact non-extremal solutions

In this section, we construct static asymptotically flat non-extremal black holes for a model with one scalar field (dilaton) non-minimally coupled to a gauge field and a non-trivial dilaton potential. We consider an arbitrary exponential dilaton coupling of the gauge field. We obtain a class of solutions that generalize the Gibbons–Maeda solutions [3] in AdS.

3.1. Generalized Gibbons–Maeda solution in AdS

Gibbons and Maeda [3] found the black hole of the Einstein–Maxwell-dilaton theory defined by the action

$$I[g_{\mu\nu}, A_\mu, \phi] = \int d^4x\sqrt{-g}\left(R - \frac{1}{4}e^{\gamma\phi}F^2 - \frac{1}{2}(\partial\phi)^2\right) \tag{8}$$

which matches the action (1) when the dilaton potential vanishes.

First step in our analysis is to rewrite the solutions of [3] in the coordinates similar with the ones we use. This is useful to make the contact with the old known solutions and also helps to gain some intuition to understanding the general class of solutions we will present.

For simplicity, we set $\gamma = (\frac{\nu+1}{\nu-1})^{1/2}$, then the Gibbons–Maeda solution is

$$ds^2 = \Omega(x)\left(-f(x)\,dt^2 + \frac{\eta^2\,dx^2}{f(x)} + d\Sigma_1\right) \tag{9}$$

$$F(x) = \frac{\eta^2 x^2(x^\nu - 1)^2 k}{x^\nu \nu^2} - \frac{\eta^2 x^2(x^\nu - 1)^3 Q^2}{2\nu^3(\nu - 1)x^{2\nu}} \tag{10}$$

with the gauge field and scalar

$$F = \frac{Q}{x^{\nu+1}}\,dx \wedge dt \qquad \phi = l_\nu^{-1}\ln(x) \tag{11}$$

The functions $\Omega(x)$ and l_ν are given in the previous section. Since the Gibbons–Maeda solution is asymptotically flat, k is fixed to 1 so that the horizon topology is spherical. With this parameterization, it is possible to consider positive or negative γ depending if one consider $x > 1$ or $x < 1$. In both cases, $x = 1$ defines the asymptotic region of the spacetime as can be seen by the pole of order two.

We would like to point out that the new parameter ν labels different solutions, for example $\nu = -1$, $\gamma = 0$, corresponds to Reissner–Nordström black hole and for $\nu > 1$ we obtain 'hairy' solutions. Therefore, the Gibbons–Maeda solution include the Reissner–Nordström solution within its parameter space, namely when $\nu = -1$. Indeed, in this case $\gamma = 0$, $\phi = 0$, and the change of coordinates

$$x = 1 + \frac{1}{\eta\rho} \tag{12}$$

brings the solution to the well-known form

$$ds^2 = -h(\rho)\,dt^2 + \frac{d\rho^2}{h(\rho)} + \rho^2\,d\Sigma_1 \tag{13}$$

where $h(\rho) = k - \frac{2M}{\rho} + \frac{q^2}{\rho^2}$ and the parameters of the two solutions are related by $\eta = -\frac{1}{2M}(k + \frac{Q^2}{4})$ and $q = \frac{Q}{2\eta}$.

What is interesting is that even in the presence of a non-trivial dilaton potential, we can still obtain asymptotically flat solutions [21]. We also present this step because it is useful for generating more general AdS solutions. We consider the following non-trivial self-interaction of the scalar field

$$V_\alpha(\phi)=2\alpha\left[\frac{\nu-1}{\nu+2}\sinh\bigl(\phi l_\nu(\nu+1)\bigr)-\frac{\nu+1}{\nu-2}\sinh\bigl(\phi l_\nu(\nu-1)\bigr)+4\frac{\nu^2-1}{\nu^2-4}\sinh(\phi l_\nu)\right] \tag{14}$$

for which there still exists an asymptotically flat black hole solution for $k=1$:

$$f_\alpha(x)=\frac{\eta^2x^2(x^\nu-1)^2k}{x^\nu\nu^2}-\frac{\eta^2x^2(x^\nu-1)^3Q^2}{2\nu^3(\nu-1)x^{2\nu}}+\alpha\left[\frac{\nu^2}{\nu^2-4}+\left(-x^2+\frac{x^{\nu+2}}{\nu+2}-\frac{x^{2-\nu}}{\nu-2}\right)\right] \tag{15}$$

Here we keep the general expression with k arbitrarily because that is what we need to write down the asymptotically AdS solutions. In flat space, we should fix $k=1$, which corresponds to the spherical horizon topology. It is clear that the Gibbons–Maeda solution can be obtained for $\alpha=0$. The role of the subindex α in the scalar field potential and metric function is only to simplify the notation in what follows.

Now we are ready for the last step. That is, we generalize the solution (15) in the presence of a negative cosmological constant. After lengthy computations we obtain:

$$f_{\alpha,\Lambda}(x)=f_\alpha-\frac{\Lambda}{3} \tag{16}$$

and the dilaton potential is

$$V_{\alpha,\Lambda}(\phi)=V_\alpha(\phi)+\frac{\Lambda(\nu^2-4)}{3\nu^2}\left[\frac{\nu-1}{\nu+2}\exp\bigl(-\phi l_\nu(\nu+1)\bigr)+\frac{\nu+1}{\nu-2}\exp\bigl(\phi l_\nu(\nu-1)\bigr)+4\frac{\nu^2-1}{\nu^2-4}\exp(-\phi l_\nu)\right] \tag{17}$$

One can check that the metric is asymptotically AdS by computing the Ricci scalar that is at the boundary $R|_{x=1}=4\Lambda$. Also, one can easily check that at the boundary $x=1$ where the dilaton vanishes the dilaton potential is $V(0)=2\Lambda$.

Furthermore, these solutions can be generalized for an electromagnetic field

$$F=\frac{Q}{x^{\nu+1}}\,dx\wedge dt+P\,dy\wedge d\varphi \tag{18}$$

with the following metric function and dilaton potential

$$f_{\alpha,\lambda,P}(x)=f_{\alpha,\lambda}(x)+\frac{P^2\eta^4x^4}{\nu^2(\nu+2)}\left(\frac{1}{4}-\frac{2x^\nu}{\nu+4}+\frac{x^{2\nu}}{2(\nu+2)}\right) \tag{19}$$

$$\begin{aligned}V_{\alpha,\lambda,P}(\phi)={}&V_{\alpha,\lambda}(\phi)+\frac{P^2\eta^4x^3(\nu+1)}{\nu^4(\nu+2)(\nu+4)}\Biggl(-\frac{(\nu+4)e^{-\nu l_\nu\phi}}{2(\nu+1)}\\&+\frac{2(\nu+8)e^{-2\nu l_\nu\phi}}{(\nu+2)}\\&-\frac{\nu^3-10\nu^2+48\nu+96}{4(2+\nu)}e^{-3\nu l_\nu\phi}\\&+\bigl(-\nu^2+3\nu+8\bigr)e^{-4\nu l_\nu\phi}\\&-\frac{(\nu+4)(\nu+2)}{4}e^{-5\nu l_\nu\phi}\Biggr)\end{aligned} \tag{20}$$

where $\lambda=\Lambda+\frac{3\eta^2P^2}{4(\nu+4)(\nu+2)^2}$.

3.2. Examples

We present examples when $\gamma=\sqrt{3}$ and $\gamma=1$ for which the potential simplifies drastically. We consider only the branches for which the dilaton is positive and so $x>1$. Interestingly enough, these solutions are smoothly connected with some solutions that can be embedded in string theory. We are able to recover some well-known solutions [6,7] in some special limits and obtain a new dyonic solution in the $\gamma=1$ case that can be embedded in string theory.

3.2.1. $\gamma=\sqrt{3}$, $\nu=2$

For this value of the coupling, we are able to obtain only $P=0$ case. The dyonic solution is technically difficult to obtain because the dilaton potential is complicated when P is turned on.

The metric is again characterized by the same conformal factor $\Omega(x)$ and the other metric function is

$$f(x)=-\frac{\Lambda}{3}-\frac{\eta^2Q^2(x^2-1)^3}{16x^2}+\alpha\left(-\frac{x^4}{12}+\frac{x^2}{3}-\frac{1}{4}-\frac{1}{3}\ln(x)\right)+\frac{k\eta^2(x^2-1)^2}{4} \tag{21}$$

The dilaton potential simplifies in this case and becomes

$$V(\phi)=2\Lambda\cosh(\phi l_2)+\frac{\alpha}{2}\left[4\phi l_2\cosh(\phi l_2)-3\sinh(\phi l_2)-\frac{1}{3}\sinh(3\phi l_2)\right] \tag{22}$$

When $\Lambda=\alpha=0$ we obtain the electrically charged KK black hole that can be embedded in $N=2$ SUGRA. In flat space the extremal limit and BPS limit coincide and a naked singularity is obtained. When only $\alpha=0$ and $k=\{0,1\}$ we obtain the AdS solutions discussed in detail in [7] but in a different coordinate system, though we also have the exact solution for $k=-1$. The non-extremal solution for $k=1$ is a regular black hole, but a domain wall for $k=\{0,-1\}$ when $\alpha=0$. In AdS the extremal limit and BPS limit do not coincide. However, we are going to present an argument in the next section that shows that the corresponding zero temperature limit is still a naked singularity.

Interestingly enough, once we consider the α-part in the potential, we obtain regular non-extremal black hole solutions with spherical, toroidal, and hyperbolic horizon topologies corresponding to $k=\{-1,0,1\}$. In this case, in the extremal limit we obtain regular zero temperature regular black holes with a finite horizon area.

3.2.2. $\gamma=1$

This limit is more subtle and it was explained for asymptotically flat black holes in [21] and we do not want to present the details here.

In this case the metric is (the form of the metric is slightly different than the general ansatz (2) in Section 2):

$$ds^2=\Omega(x)\left[-f(x)\,dt^2+\frac{\eta^2\,dx^2}{x^2f(x)}+d\Sigma_k\right] \tag{23}$$

where

$$f(x)=-\frac{\Lambda}{3}+\alpha\left[\frac{(x^2-1)}{2x}-\ln(x)\right]+k\eta^2\frac{(x-1)^2}{x}+\frac{\eta^2}{2x^2}(x-1)^3\bigl(P^2\eta^2x-Q^2\bigr) \tag{24}$$

The gauge field and dilaton potential are

$$F = \frac{Q}{x^2}\,dx \wedge dt + P\,dy \wedge d\varphi \tag{25}$$

$$V(\phi) = \left(\frac{\Lambda}{3} + \alpha\phi\right)\left(4 + 2\cosh(\phi)\right) - 6\alpha\sinh(\phi) \tag{26}$$

When $\Lambda = \alpha = 0$ the solution is asymptotically flat and can be embedded in $N = 4$ SUGRA. When only $\alpha = 0$ the situation is similar with the one in $\gamma = \sqrt{3}$ case and the black hole/domain wall solutions can be embedded in string theory. The extremal limit when $\alpha \neq 0$ is discussed in detail in the next section.

4. Extremal limit and attractor mechanism

As we have already mentioned, when α is turned on, we are going to show tat the extremal limit is a regular zero-temperature black hole with finite horizon area even when $P = 0$. This is possible due to the existence of a non-trivial dilaton potential. In this case, there is a modified effective potential [16]:

$$V_{eff}(\phi) = \frac{1}{b^2}\left[e^{-\phi}Q^2 + e^{\phi}P^2\right] + 2b^2 V(\phi) \tag{27}$$

that controls the dilaton flow.

When $P = 0$ and $V(\phi) = constant$ this effective potential cannot have a minimum and so there no exist regular extremal black hole with finite horizon area.[3] When the dilaton potential is non-trivial $V(\phi) \neq 0$, it could be possible that the effective potential has a minimum at the horizon and in the extremal limit one can obtain a black hole with finite entropy.

Since the effective potential method and entropy function formalism are equivalent in the near horizon limit,[4] we prefer to use the entropy function of Sen [17][5] to prove the existence of regular extremal black holes with finite horizon area when $P = 0$.

The near horizon geometry of 4-dimensional static extremal black holes has an enhanced symmetry of $AdS_2 \times S^2$, and unlike the non-extremal case, it is a solution of the equations of motion. We consider the following ansatz for the metric and gauge field:

$$ds^2 = v_1\left(-r^2\,dt^2 + \frac{dr^2}{r^2}\right) + v_2\left[\frac{dy^2}{1-y^2} + \left(1-y^2\right)d\phi^2\right]$$
$$F = E\,dx \wedge dt \tag{28}$$

The entropy function is related to the action evaluated at the horizon by:

$$\begin{aligned} &S(E, v_1, v_2, u) \\ &\quad = 2\pi(Eq - I_H) = 2\pi\left[Eq - \left(-2v_2 + 2v_1 - v_1 v_2 V(u) \right.\right. \\ &\qquad \left.\left. + 2^{-1}v_1^{-1}v_2 e^u E^2\right)\right] \end{aligned} \tag{29}$$

where q is the physical charge and u is the value of the scalar field at the horizon. The attractor equations are

$$\frac{\partial S}{\partial E} = q - \frac{v_2}{v_1}e^u E = 0 \tag{30}$$

$$\frac{\partial S}{\partial v_1} = 2 - v_2 V(u) - 2^{-1}v_1^{-2}v_2 e^u E^2 = 0 \tag{31}$$

$$\frac{\partial S}{\partial v_2} = -2 - v_1 V(u) + 2^{-1}v_1^{-1}e^u E^2 = 0 \tag{32}$$

$$\frac{\partial S}{\partial u} = -v_1 v_2 V'(u) + 2^{-1}v_1^{-1}v_2 e^u E^2 = 0 \tag{33}$$

Eq. (30) can be easily solved and then find v_1 and v_2 as functions of the physical charge and the value of the dilaton at the horizon. A further manipulation of the equations allows to implicitly find the value of the scalar at the horizon for $q^2 \neq 0$

$$q^2\left(V' + V\right)^2 = 8e^u V' \tag{34}$$

Let us consider now (26) and replace it in (34) — for $\alpha = 0$ we get

$$\frac{q^2}{l^2} = -\frac{4e^u \sinh u}{(\sinh u + \cosh u + 2)^2} \tag{35}$$

which has real solutions only when $u < 0$, which corresponds to the rank of coordinates $x < 1$. Since we are interested in the $x > 1$ branch we conclude, as expected, that the extremal limit is a naked singularity. However, when $\alpha \neq 0$ Eq. (34) yields

$$\frac{q^2}{l^2} = \frac{4e^u(2\alpha l^2 + \alpha l^2 u \sinh u - 2\alpha l^2 \cosh u - \sinh u)}{[2 + \sinh u + \cosh u + \alpha l^2(2\cosh u - u\cosh u - u\sinh u - 2u + 3\sinh u - 2)]^2} \tag{36}$$

Eq. (36) is a quadratic equation for α. It turns out that the discriminant of this equation is always positive, which indicates the existence of real solutions for any values of q, l, and u.

5. Discussion and future directions

In this Letter, we have constructed a general class of four-dimensional AdS dyonic black holes that generalize the black hole solutions of Gibbons and Maeda in flat space [3]. The dilaton potential has two parts, one that is controlled by the cosmological constant and the other one controlled by an arbitrary parameter α. Obviously, since the solutions are asymptotically AdS, the part controlled by α does not play any role at the boundary but controls the IR regime of the dual field theory. This is why we find well defined moduli flows for the corresponding extremal black holes even if just one gauge field (with only one charge) is turned on. Since the extremal (attractor) horizons are infinitely far away in the bulk they do not get distorted by changing the scalars boundary values and so, from this point of view, the attractor mechanism acts as a no-hair theorem for extremal black holes [20].

One can speculate that if the quantum corrections (which could be similar with the α-part of the potential) become relevant and the dilaton potential gets corrected, naked singularities can get dressed with a horizon and in this case the IR point of the RG flow is well defined. Sure, it is not clear if our solutions can be embedded in a fundamental theory like string theory for $\alpha \neq 0$, but the main point is that these are generic features and a similar situation may exist for 'stringy' solutions.

When $\alpha = 0$ we were able to recover some known solutions that can be embedded in string theory [5,7]. In this particular case, we can explicitly obtain the superpotential associated with the dilaton potential. The solution becomes a domain wall in the planar case $k = 0$, but in the spherical case $k = 1$ we can still obtain regular black hole solutions. The superpotential equation is

$$V(\phi) = 2\left(\frac{dW(\phi)}{d\phi}\right)^2 - \frac{3}{2}W(\phi)^2 \tag{37}$$

and when $\alpha = 0$ we obtain

[3] When a second charge is turned on, the effective potential has a minimum and regular solutions exist.

[4] Since we have exact solutions, we could find the near horizon directly for each solution. However, such a computation is difficult and, particularly, not very illuminating.

[5] The entropy function formalism for AdS black holes was initiated in [23] and a detailed analysis of entropy function and near horizon data for AdS black hole can be found in [24].

$$W(\phi) = l^{-1}\left[\frac{(\nu+1)}{\nu}\exp\left(\left(\frac{\nu-1}{2}\right)\phi l_\nu\right) + \frac{(\nu-1)}{\nu}\exp\left(-\left(\frac{\nu+1}{2}\right)\phi l_\nu\right)\right] \quad (38)$$

where it was set $\Lambda = -\frac{3}{l^2}$.

Let us end up this section by presenting a few future directions. Since we have the superpotential, we can construct the corresponding RG flow as in [25]. Also an analysis similar with the one of [26] in the context of fake supergravity can be done. A generalization of these solutions to 5 dimensions is possible and we have some preliminary results [27]. Our solutions can also be useful in the context of the recent work [28].

In [7] an intriguing thermodynamic feature of the gauged dyonic black hole was pointed out. That is, the usual first-law of thermodynamics does hold just in some special cases. However, in general, a new pair of thermodynamic conjugate variables should appear in the first law. They describe the potential and charge of the scalar hair that breaks some of the asymptotic AdS symmetries. A similar analysis can be also done for our solutions.

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