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Simplified pure spinor b ghost in a curved heterotic superstring background

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Abstract: Using the RNS-like fermionic vector variables introduced in arXiv:1305.0693, the pure spinor b ghost in a curved heterotic superstring background is easily constructed. This construction simplifies and completes the b ghost construction in a curved background of arXiv:1311.7012.

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1 Introduction

Although the pure spinor formalism for the superstring [1] has several nice features such as manifest spacetime supersymmetry and a simple BRST operator, a complicated feature of the formalism which needs to be better understood is the pure spinor b ghost. Unlike in the bosonic string or Ramond-Neveu-Schwarz (RNS) string where the b ghost is a fundamental worldsheet variable, the b ghost in the pure spinor formalism is a composite operator constructed from the other worldsheet variables.

The BRST operator Q in the pure spinor formalism is independent of the target-space background and takes the simple form

$$Q = \int dz \, (\lambda^\alpha d_\alpha + \widehat{\omega}^\alpha r_\alpha) \tag{1.1}$$

where $(\lambda^\alpha, d_\alpha, \widehat{\omega}^\alpha, r_\alpha)$ are fundamental worldsheet variables. To satisfy the relation $\{Q, b\} = T$ where T is the conformal stress tensor, b was constructed in a flat background in [2] as a complicated function of the worldsheet fields. This construction of the pure spinor b ghost was simplified in [3] by introducing a fermionic vector field $\overline{\Gamma}_a$ which was related to the usual RNS fermionic vector field ψ_a by "dynamical twisting".

An important question is how to generalize this construction of the pure spinor b ghost in a curved superstring background. In [4], the pure spinor b ghost was constructed in a super-Maxwell background of the open superstring, and in [5], the pure spinor b ghost was constructed in an N=1 supergravity background of the heterotic superstring. The construction in [5] for a curved heterotic superstring background was quite complicated and some of the coefficients of the composite operator for the pure spinor b ghost were not explicitly computed.

In this paper, we will use the RNS-like fermionic vector field $\overline{\Gamma}_a$ of [3] to simplify the construction of the pure spinor b ghost in a curved heterotic superstring background. When expressed in terms of $\overline{\Gamma}_a$, the composite operator for the pure spinor b ghost in a curved heterotic background is a simple covariantization of the composite operator in a flat background that was found in [3]. In addition to simplifying the construction in a curved heterotic background, it is hoped that this method involving $\overline{\Gamma}_a$ will also be useful for constructing the pure spinor b ghost in an N=2 supergravity background of the Type II superstring.

In section 2, we review the pure spinor formalism of the heterotic superstring in a flat and curved target-superspace background. In section 3, we define the RNS-like variable $\overline{\Gamma}_a$ in a curved background. And in section 4, we use $\overline{\Gamma}_a$ to explicitly construct the pure spinor b ghost in a curved heterotic superstring background. The appendix computes the BRST transformations of $\overline{\Gamma}_a$ and the b ghost in a flat background.

2 Review of the non-minimal pure spinor formalism

In this section we review the non-minimal pure spinor formalism of the heterotic superstring in flat space [2] and in a curved background [5].

2.1 Non-minimal pure spinor formalism in flat space

The non-minimal pure spinor formalism in the heterotic superstring is constructed using the ten-dimensional $N = 1$ superspace coordinates (X^a, θ^α) for $a = 0$ to 9 and $\alpha = 1$ to 16, the conjugate variable of θ^α which is called p_α, a set of pure spinor variables $\left(\lambda^\alpha, \widehat{\lambda}_\alpha, r_\alpha\right)$ together with their conjugate variables $(\omega_\alpha, \widehat{\omega}^\alpha, s^\alpha)$, and the same 32 fermionic right-moving variables ξ^R for $R = 1$ to 32 as in the RNS heterotic superstring formalism. The pure spinor variables are constrained to satisfy

$$(\lambda\gamma^a\lambda) = \left(\widehat{\lambda}\gamma^a\widehat{\lambda}\right) = \left(\widehat{\lambda}\gamma^a r\right) = 0\,, \tag{2.1}$$

where $(\gamma^a)_{\alpha\beta}$ and $(\gamma^a)^{\alpha\beta}$ are the symmetric gamma matrices which satisfy the Dirac algebra

$$(\gamma^a)_{\alpha\gamma}\left(\gamma^b\right)^{\gamma\beta} + \left(\gamma^b\right)_{\alpha\gamma}(\gamma^a)^{\gamma\beta} = 2\eta^{ab}\delta_\alpha^\beta\,. \tag{2.2}$$

Because of the pure spinor conditions (2.1), the conjugate pure spinor variables are defined up to the gauge invariances

$$\delta\omega_\alpha = (\lambda\gamma^a)_\alpha\,\Lambda_{1a}, \quad \delta s^\alpha = \left(\gamma^a\widehat{\lambda}\right)^\alpha\Lambda_{2a}, \quad \delta\widehat{\omega}^\alpha = \left(\gamma^a\widehat{\lambda}\right)^\alpha\Lambda_{3a} - (\gamma^a r)\,\Lambda_{2a}\,, \tag{2.3}$$

where $\Lambda_1, \Lambda_2, \Lambda_3$ are arbitrary gauge parameters. The action of the theory is quadratic in these variables and is given by

$$S = S_0 + \int d^2z\left(\widehat{\omega}^\alpha\overline{\partial}\widehat{\lambda}_\alpha + s^\alpha\overline{\partial}r_\alpha\right)\,, \tag{2.4}$$

where

$$S_0 = \int d^2z\left(\frac{1}{2}\partial X^a\overline{\partial}X_a + p_\alpha\overline{\partial}\theta^\alpha + \omega_\alpha\overline{\partial}\lambda^\alpha + \xi^R\partial\xi^R\right) \tag{2.5}$$

is the minimal pure spinor action.

The quantization of this system is performed by introducing a left-moving BRST charge given by

$$Q = \oint dz(\lambda^\alpha d_\alpha + \widehat{\omega}^\alpha r_\alpha)\,, \tag{2.6}$$

where

$$d_\alpha = p_\alpha - \frac{1}{2}(\theta\gamma^a)_\alpha \left(\partial X_a + \frac{1}{4}(\theta\gamma_a\partial\theta)\right)\,. \tag{2.7}$$

This BRST charge is nilpotent because of the OPE

$$d_\alpha(y)d_\beta(z) \to -\frac{1}{(y-z)}\gamma^a_{\alpha\beta}\Pi_a(z) \tag{2.8}$$

where

$$\Pi_a = \partial X_a + \frac{1}{2}(\theta\gamma_a\partial\theta)\,. \tag{2.9}$$

The BRST transformations of the worldsheet fields of our system are

$$\begin{aligned} &Q\Pi^a = -(\lambda\gamma^a\partial\theta)\,, \quad Q\theta^\alpha = \lambda^\alpha\,, \quad Qd_\alpha = \Pi^a(\gamma_a\lambda)_\alpha\,, \quad Q\lambda^\alpha = 0\,, \quad Q\omega_\alpha = d_\alpha\,, \\ &Q\widehat{\lambda}_\alpha = -r_\alpha\,, \qquad Q\widehat{\omega}^\alpha = 0\,, \qquad Qr_\alpha = 0\,, \qquad Qs^\alpha = \widehat{\omega}^\alpha\,. \end{aligned} \tag{2.10}$$

Note that the non-minimal sector in our system does not change the cohomology of the minimal sector and the action (2.4) can be written as

$$S = S_0 + Q\int d^2z\; s^\alpha\overline{\partial}\widehat{\lambda}_\alpha\,. \tag{2.11}$$

The non-minimal pure spinor formalism does not contain the (b,c) worldsheet reparameterization ghosts. However, one can construct an operator b satisfying the equation $Qb = T$ where T is the world-sheet stress tensor of the action (2.4), and this operator is identified with the pure spinor b ghost [2]. It was shown in [3] that this b ghost is simplified by introducing the RNS-like fermionic vector

$$\overline{\Gamma}_a = -\frac{1}{2\left(\lambda\widehat{\lambda}\right)}\left(d\gamma_a\widehat{\lambda}\right) - \frac{1}{8\left(\lambda\widehat{\lambda}\right)^2}\left(r\gamma_{abc}\widehat{\lambda}\right)N^{bc} \tag{2.12}$$

where $N^{ab} = \frac{1}{2}(\lambda\gamma^{ab}\omega)$ is the Lorentz current for the minimal pure spinors.

In terms of (2.12), the pure spinor b ghost is

$$b = -s^\alpha\partial\widehat{\lambda}_\alpha - \omega_\alpha\partial\theta^\alpha + \Pi^a\overline{\Gamma}_a - \frac{1}{4\left(\lambda\widehat{\lambda}\right)}\left(\lambda\gamma^{ab}r\right)\overline{\Gamma}_a\overline{\Gamma}_b + \frac{1}{2\left(\lambda\widehat{\lambda}\right)}\left(\omega\gamma_a\widehat{\lambda}\right)(\lambda\gamma^a\partial\theta)\,. \tag{2.13}$$

To verify the relation $Qb = T$, we first compute the action of Q on $\overline{\Gamma}_a$ to be

$$Q\overline{\Gamma}_a = -\frac{1}{2\left(\lambda\widehat{\lambda}\right)}\Pi^b\left(\widehat{\lambda}\gamma_a\gamma_b\lambda\right) - \frac{1}{4\left(\lambda\widehat{\lambda}\right)^2}(\lambda\gamma_{bc}r)\left(\widehat{\lambda}\gamma^c\gamma_a\lambda\right)\overline{\Gamma}^b\,. \tag{2.14}$$

The proof of (2.14) and the verification of $Qb = T$ in a flat background are in the appendix.

The purpose of this paper is to find a $\overline{\Gamma}_a$ in a curved heterotic superstring background that satisfies an equation analogous to (2.14) and to define the b ghost as the covariantization of (2.13). In order to do this, we will need the BRST transformations corresponding to (2.10) in a curved background. We now review the non-minimal pure spinor formalism in a curved heterotic superstring background.

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2.2 Non-minimal pure spinor formalism in a curved background

The minimal sector of the heterotic string in a curved background was constructed in [6]. The action has the form

$$S_0 = \int d^2 z \left(\frac{1}{2}\Pi_a \overline{\Pi}^a + \frac{1}{2}\Pi^A \overline{\Pi}^B B_{BA} + d_\alpha \overline{\Pi}^\alpha + \omega_\alpha \overline{\nabla}\lambda^\alpha + \xi^R \nabla \xi^R + \frac{1}{2}\alpha' \Phi r \right), \tag{2.15}$$

where Π^A and $\overline{\Pi}^A$ for $A = (a, \alpha)$ are defined from the background supervielbein $E_M{}^A$ and the target superspace coordinates Z^M as $\Pi^A = \partial Z^M E_M{}^A$ and $\overline{\Pi}^A = \overline{\partial} Z^M E_M{}^A$, B_{BA} is the graded-antisymmetric two-form superfield, Φ is the dilaton superfield which couples to the two-dimensional worldsheet curvature r,

$$\nabla \xi^R = \partial \xi^R + T_I^{RS} \xi^S \left(\Pi^A A_A^I + d_\alpha W^{I\alpha} + \frac{1}{2} N_{ab} F^{Iab} \right) \tag{2.16}$$

where T_I^{RS} are the SO(32) adjoint matrices for $I = 1$ to 496 and $\left(A_M^I, W^{I\alpha}, F^{Iab}\right)$ are the super-Yang-Mills gauge fields and field-strengths, and

$$\overline{\nabla}\lambda^\alpha = \overline{\partial}\lambda^\alpha + \lambda^\beta \overline{\Pi}^A \Omega_{A\beta}{}^\alpha \tag{2.17}$$

where the connection $\Omega_{A\beta}{}^\alpha$ has the structure

$$\Omega_{A\beta}{}^\alpha = \Omega_A \delta_\beta^\alpha + \frac{1}{4}\Omega_{Aab} \left(\gamma^{ab}\right)_\beta{}^\alpha . \tag{2.18}$$

Here Ω_{Aab} is the usual Lorentz connection and Ω_A is a connection for scaling transformations introduced in [6], and one can verify their coupling preserves the pure spinor gauge invariance of (2.3).

The presence of the scale connection Ω_A in (2.15) implies that the action is invariant not only under the usual local Lorentz transformations, but also under the local fermionic scale transformations

$$\begin{aligned} &\delta\lambda^\alpha = \Lambda\lambda^\alpha , \quad \delta\omega_\alpha = -\Lambda\omega_\alpha , \qquad \delta d_\alpha = -\Lambda d_\alpha , \\ &\delta\Omega_{\alpha\beta}{}^\gamma = -(\partial_\alpha \Lambda)\delta_\beta^\gamma - \Lambda \Omega_{\alpha\beta}{}^\gamma , \qquad \delta\Omega_{a\beta}{}^\gamma = -(\partial_a\Lambda)\delta_\beta^\gamma , \\ &\delta E_M{}^\alpha = \Lambda E_M{}^\alpha , \qquad \delta E_\alpha{}^M = -\Lambda E_\alpha{}^M . \end{aligned} \tag{2.19}$$

So variables and superfields with raised tangent-space spinor indices carry charge +1 with respect to the fermionic scale transformations and variables and superfields with lowered tangent-space spinor indices carry charge −1.

The minimal BRST charge is given by $Q_0 = \oint \lambda^\alpha d_\alpha$ and it was shown in [6] that nilpotency and holomorphicity of Q_0 forces the background to satisfy the equations of $N = 1$ ten-dimensional supergravity. Nilpotency implies that

$$\lambda^\alpha \lambda^\beta T_{\alpha\beta}{}^A = 0 , \quad \lambda^\alpha\lambda^\beta\lambda^\gamma R_{\alpha\beta\gamma}{}^\delta = 0 , \tag{2.20}$$

where $T_{\alpha\beta}{}^A$ and $R_{\alpha\beta\gamma}{}^\delta$ are torsion and curvature components. And as shown in [6], nilpotency and holomorphicity imply that the torsion components can be gauge-fixed to the form

$$T_{\alpha\beta}{}^a = \gamma_{\alpha\beta}^a , \quad T_{A\beta}{}^\gamma = 0 , \quad T_{\alpha a}{}^b = 2(\gamma_a{}^b)_\alpha{}^\beta \Omega_\beta . \tag{2.21}$$

In addition, the absence of chiral [6] and conformal [7] anomalies of the worldsheet action implies that Ω_α is related to the dilaton superfield Φ by

$$\Omega_\alpha = \frac{1}{4} D_\alpha \Phi \,. \tag{2.22}$$

The BRST transformations of the minimal fields in (2.15) were determined in [8] to be

$$\begin{aligned} Q\Pi^a &= -\lambda^\beta \Omega_{\beta b}{}^a \Pi^b - \lambda^\alpha \Pi^b T_{b\alpha}{}^a \,, & Q\Pi^\alpha &= -\lambda^\beta \Omega_{\beta\gamma}{}^\alpha \Pi^\gamma + \nabla\lambda^\alpha \,, \\ Q\lambda^\alpha &= -\lambda^\beta \Omega_{\beta\gamma}{}^\alpha \lambda^\gamma \,, & Q\omega_\alpha &= \lambda^\beta \Omega_{\beta\alpha}{}^\gamma \omega_\gamma + d_\alpha \,, \\ Qd_\alpha &= \lambda^\beta \Omega_{\beta\alpha}{}^\gamma d_\gamma + \Pi^a (\gamma_a \lambda)_\alpha + \lambda^\beta \lambda^\gamma \omega_\delta R_{\alpha\beta\gamma}{}^\delta \,, \end{aligned} \tag{2.23}$$

where the first term in these transformations is a Lorentz and scale transformation proportional to $\lambda^\beta \Omega_{\beta\gamma}{}^\alpha$.

For the non-minimal sector, it was noted in [5] that there is an effect of the background geometry on the BRST transformations of the non-minimal pure spinor fields. Assuming that the minimal sector is unaffected by the non-minimal variables, a cohomological argument determined that $\left(\widehat{\lambda}, \widehat{\omega}, r, s\right)$ transform in a curved background as

$$\begin{aligned} Q\widehat{\lambda}_\alpha &= -r_\alpha + \lambda^\gamma \widehat{\lambda}_\beta \left(\Omega_{\gamma\alpha}{}^\beta - \frac{1}{4} T_{\gamma ab} \left(\gamma^{ab}\right)_\alpha{}^\beta \right), \\ Q\widehat{\omega}^\alpha &= -\widehat{\omega}^\beta \lambda^\gamma \left(\Omega_{\gamma\beta}{}^\alpha - \frac{1}{4} T_{\gamma ab} \left(\gamma^{ab}\right)_\beta{}^\alpha \right), \\ Qs^\alpha &= \widehat{\omega}^\alpha + s^\beta \lambda^\gamma \left(\Omega_{\gamma\beta}{}^\alpha - \frac{1}{4} T_{\gamma ab} \left(\gamma^{ab}\right)_\beta{}^\alpha \right), \\ Qr_\alpha &= -\lambda^\gamma r_\beta \left(\Omega_{\gamma\alpha}{}^\beta - \frac{1}{4} T_{\gamma ab} \left(\gamma^{ab}\right)_\alpha{}^\beta \right). \end{aligned} \tag{2.24}$$

Note that the torsion $T_{\gamma ab}$ includes the Lorentz connection $\Omega_{\gamma ab}$, so the Lorentz part of the spin connection of (2.18) does not appear in these non-minimal BRST transformations.

To construct the non-minimal action in a curved background, the BRST-trivial term

$$S_{\text{non-min}} = Q \int d^2 z \left(s \overline{\nabla} \widehat{\lambda} + \frac{1}{4} \overline{\Pi}^A T_{Aab} \left(s \gamma^{ab} \widehat{\lambda} \right) \right) \tag{2.25}$$

will be added to the minimal action of (2.15) where $\overline{\nabla} \widehat{\lambda}_\alpha = \overline{\partial} \widehat{\lambda}_\alpha - \widehat{\lambda}_\beta \overline{\Pi}^A \Omega_{A\alpha}{}^\beta$. This construction is analogous to the flat action of (2.11), and although the torsion term in (2.25) is not needed for covariance and was not included in [5], it will simplify the construction by decoupling the Lorentz connection Ω_{Aab} from the non-minimal action. Using the BRST transformations of (2.24), one finds that

$$\begin{aligned} S_{\text{non-min}} = & \int d^2 z \left(\widehat{\omega}^\alpha \overline{\nabla} \widehat{\lambda}_\alpha + s^\alpha \overline{\nabla} r_\alpha + \frac{1}{4} \overline{\Pi}^A T_{Aab} \left(\widehat{\omega} \gamma^{ab} \widehat{\lambda} + s \gamma^{ab} r \right) \right. & (2.26) \\ & \left. + \lambda^\alpha \overline{\Pi}^A R_{A\alpha} \left(s^\beta \widehat{\lambda}_\beta \right) + \frac{1}{4} \lambda^\alpha \overline{\Pi}^A \left(R_{A\alpha ab} - \nabla_{[A} T_{\alpha]ab} - T_{A\alpha}{}^c T_{cab} + T_{Ac[a} T_{b]\alpha}{}^c \right) \left(s \gamma^{ab} \widehat{\lambda} \right) \right) \\ = & \int d^2 z \left(\widehat{\omega}^\alpha \overline{\nabla} \widehat{\lambda}_\alpha + s^\alpha \overline{\nabla} r_\alpha + \frac{1}{4} \overline{\Pi}^A T_{Aab} \left(\widehat{\omega} \gamma^{ab} \widehat{\lambda} + s \gamma^{ab} r \right) \right. & (2.27) \\ & \left. + \lambda^\alpha \overline{\Pi}^A R_{A\alpha} \left(s^\beta \widehat{\lambda}_\beta \right) + \frac{1}{4} \lambda^\alpha \overline{\Pi}^d \left(R_{d\alpha ab} - \nabla_{[d} T_{\alpha]ab} - T_{d\alpha}{}^c T_{cab} + T_{dc[a} T_{b]\alpha}{}^c \right) \left(s \gamma^{ab} \widehat{\lambda} \right) \right) \end{aligned}$$

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where we have used the Bianchi identity

$$R_{\beta\alpha ab} - \nabla_{(\beta}T_{\alpha)ab} - T_{(\beta a}{}^{c}T_{\alpha)cb} - \gamma^{c}_{\beta\alpha}T_{cab} = 0 \tag{2.28}$$

in the second line of (2.26).

Using the Noether method, one can easily determine the BRST charge corresponding to the action of $S = S_0 + S_{\text{non-min}}$ to be

$$Q = \int dz\, (\lambda^\alpha d_\alpha + \widehat{\omega}^\alpha r_\alpha)\,. \tag{2.29}$$

3 Definition of $\overline{\Gamma}_a$ in curved background

3.1 Simplified BRST transformations

The first step in defining the curved background generalization of $\overline{\Gamma}_a$ of (2.12) is to define a new variable

$$D_\alpha = d_\alpha + \frac{1}{4}\lambda^\beta T_{\beta ab}\left(\gamma^{ab}\omega\right)_\alpha - 3(\lambda\Omega)\omega_\alpha\,. \tag{3.1}$$

In terms of D_α, the BRST transformation of ω_α is given by

$$Q\omega_\alpha = \lambda^\beta \Omega_{\beta\alpha}{}^{\gamma}\omega_\gamma + D_\alpha + 3(\lambda\Omega)\omega_\alpha - \frac{1}{4}\lambda^\beta T_{\beta ab}\left(\gamma^{ab}\omega\right)_\alpha\,. \tag{3.2}$$

Furthermore, the BRST transformation of D_α is

$$\begin{aligned} QD_\alpha &= \lambda^\beta \Omega_{\beta\alpha}{}^{\gamma}D_\gamma + \Pi^a(\gamma_a\lambda)_\alpha + 3(\lambda\Omega)D_\alpha - \frac{1}{4}\lambda^\beta T_{\beta ab}\left(\gamma^{ab}D\right)_\alpha \\ &\quad + \lambda^\beta\lambda^\gamma\omega_\delta\left(R_{\alpha\beta\gamma}{}^{\delta} + \frac{1}{4}\left(\gamma^{ab}\right)_\alpha{}^{\delta}\nabla_\gamma T_{\beta ab} + \frac{1}{16}\left(\gamma^{ab}\gamma^{cd}\right)_\alpha{}^{\delta}T_{\beta ab}T_{\gamma cd}\right) \\ &= \lambda^\beta \Omega_{\beta\alpha}{}^{\gamma}D_\gamma + \Pi^a(\gamma_a\lambda)_\alpha + 3(\lambda\Omega)D_\alpha - \frac{1}{4}\lambda^\beta T_{\beta ab}\left(\gamma^{ab}D\right)_\alpha \end{aligned} \tag{3.3}$$

where we have used that $Q(\lambda\Omega) = 0$ because Ω_α is proportional to $\nabla_\alpha\Phi$. To prove that the second line in (3.3) is zero, symmetrize in $(\beta\gamma)$ and use the Bianchi identity $R_{(\alpha\beta\gamma)}{}^{\delta} = 0$ and $\lambda^\beta\lambda^\gamma R_{\beta\gamma} = 0$ to show that the second line is equal to

$$\begin{aligned} &\frac{1}{8}\lambda^\beta\lambda^\gamma\omega_\delta\left(\left(\gamma^{ab}\right)_\alpha{}^{\delta}\left(-R_{\beta\gamma ab} + \nabla_{(\beta}T_{\gamma)ab}\right) + \frac{1}{4}\left[\gamma^{ab},\gamma^{cd}\right]_\alpha{}^{\delta}T_{\beta ab}T_{\gamma cd}\right) \\ &\quad = \frac{1}{8}\lambda^\beta\lambda^\gamma\omega_\delta\left(\nabla_{(\beta}T_{\gamma)ab} - T_{a(\beta}{}^{c}T_{\gamma)cb} - R_{\beta\gamma ab}\right)\left(\gamma^{ab}\right)_\alpha{}^{\delta} = -\frac{1}{8}\lambda^\beta\lambda^\gamma\omega_\delta\gamma^{c}_{\beta\gamma}T_{cab}\left(\gamma^{ab}\right)_\alpha{}^{\delta} = 0\,, \end{aligned}$$

where we used the Bianchi identity for $R_{[\beta\gamma a]b}$.

The BRST transformations of (2.24), (3.2) and (3.3) all involve a Lorentz and scale transformation proportional to

$$-\lambda^\beta\Omega_{\beta\alpha}{}^{\gamma} + \frac{1}{4}\lambda^\beta T_{\beta ab}\left(\gamma^{ab}\right)_\alpha{}^{\gamma}\,. \tag{3.4}$$

It will be useful to define $\tilde{Q} = Q - Q_{L+S}$ where Q_{L+S} is this Lorentz and scale transformation, and one finds that

$$\begin{aligned} &\tilde{Q}\Pi^a = -\left(\lambda\gamma^a\Pi\right), \quad \tilde{Q}\Pi^\alpha = \nabla\lambda^\alpha + \frac{1}{4}\lambda^\beta T_{\beta ab}\left(\gamma^{ab}\Pi\right)^\alpha, \quad \tilde{Q}D_\alpha = \Pi^a\left(\gamma_a\lambda\right)_\alpha + 3(\lambda\Omega)D_\alpha, \\ &\tilde{Q}\lambda^\alpha = 5(\lambda\Omega)\lambda^\alpha, \quad \tilde{Q}\omega_\alpha = D_\alpha + 3(\lambda\Omega)\omega_\alpha, \\ &\tilde{Q}\widehat{\lambda}_\alpha = -r_\alpha, \quad \tilde{Q}\widehat{\omega}^\alpha = 0, \quad \tilde{Q}r_\alpha = 0, \quad \tilde{Q}s^\alpha = \widehat{\omega}^\alpha, \end{aligned} \tag{3.5}$$

where we have used that

$$\tilde{Q}\lambda^\alpha = \frac{1}{4}\lambda^\beta T_{\beta ab}\left(\gamma^{ab}\lambda\right)^\alpha = \frac{1}{2}(\lambda\gamma_{ab}\Omega)\left(\gamma^{ab}\lambda\right)^\alpha = 5(\lambda\Omega)\lambda^\alpha. \tag{3.6}$$

3.2 Construction of $\overline{\Gamma}_a$

In this subsection, it will be shown that

$$\overline{\Gamma}_a = -\frac{1}{2\left(\lambda\widehat{\lambda}\right)}\left(D\gamma_a\widehat{\lambda}\right) - \frac{1}{8\left(\lambda\widehat{\lambda}\right)^2}\left(r\gamma_{abc}\widehat{\lambda}\right)N^{bc} \tag{3.7}$$

satisfies the BRST transformation

$$\tilde{Q}\overline{\Gamma}_a = -\frac{1}{2\left(\lambda\widehat{\lambda}\right)}\Pi^b\left(\widehat{\lambda}\gamma_a\gamma_b\lambda\right) - \frac{1}{4\left(\lambda\widehat{\lambda}\right)^2}\left(\lambda\gamma_{bc}r\right)\left(\widehat{\lambda}\gamma^c\gamma_a\lambda\right)\overline{\Gamma}^a - 2\left(\lambda^\alpha\Omega_\alpha\right)\overline{\Gamma}_a \tag{3.8}$$

where $\tilde{Q}$ is defined in (3.5). Comparing with the equations of (2.12) and (2.14), one sees that (3.7) is constructed in a curved background by replacing d_α in the flat-space construction with D_α of (3.1).

Since the BRST transformations of (3.5) closely resemble the flat space BRST transformation of (2.10), the only necessary step to proving (3.8) is to show that the terms $(\lambda^\alpha\Omega_\alpha)$ which appear in (3.5) sum up to $-2(\lambda^\alpha\Omega_\alpha)\overline{\Gamma}_\alpha$. From the first term in (3.7), one obtains

$$\frac{1}{2\left(\lambda\widehat{\lambda}\right)^2}\left(5\left(\lambda\widehat{\lambda}\right)(\lambda\Omega)\right)\left(D\gamma_a\widehat{\lambda}\right) - \frac{1}{2\left(\lambda\widehat{\lambda}\right)}\left(3(\lambda\Omega)D_\alpha\right)\left(\gamma_a\widehat{\lambda}\right)^\alpha = 2\left(\lambda\Omega\right)\frac{1}{2\left(\lambda\widehat{\lambda}\right)}\left(D\gamma_a\widehat{\lambda}\right), \tag{3.9}$$

where the first term comes from the transformation of $\left(\lambda\widehat{\lambda}\right)^{-1}$ and the second term comes from the transformation of D_α. And from the second term in (3.7), one obtains

$$\begin{aligned} &\frac{1}{4\left(\lambda\widehat{\lambda}\right)^3}\left(5\left(\lambda\widehat{\lambda}\right)(\lambda\Omega)\right)\left(r\gamma_{abc}\widehat{\lambda}\right)N^{bc} + \frac{1}{8\left(\lambda\widehat{\lambda}\right)^2}\left(r\gamma_{abc}\widehat{\lambda}\right)\left(5(\lambda\Omega)N^{bc} + 3(\lambda\Omega)N^{bc}\right) \\ &\quad = 2(\lambda\Omega)\frac{1}{8\left(\lambda\widehat{\lambda}\right)^2}\left(r\gamma_{abc}\widehat{\lambda}\right)N^{bc} \end{aligned} \tag{3.10}$$

where the first term comes from the transformation of $\left(\lambda\widehat{\lambda}\right)^{-1}$ and the second term comes from the transformation of N^{bc}. So we have proven (3.8).

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4 Definition of b ghost in curved background

In this section, we will use the dynamical twisting method of [3] to simplify the construction of the b ghost in a curved heterotic background which was proposed in [5]. We will show that the b ghost in a curved background can be defined in terms of the dynamically twisted RNS-like variable (3.7) by simply covariantizing the flat-space expression of (2.13) as

$$b = -s^\alpha \nabla \widehat{\lambda}_\alpha + \frac{1}{4}\Pi^A T_{Aab}\left(s\gamma^{ab}\widehat{\lambda}\right) - \omega_\alpha \Pi^\alpha + \Pi^a \overline{\Gamma}_a - \frac{1}{4\left(\lambda\widehat{\lambda}\right)}\left(\lambda\gamma^{ab}r\right)\overline{\Gamma}_a\overline{\Gamma}_b + \frac{1}{2\left(\lambda\widehat{\lambda}\right)}\left(\omega\gamma_a\widehat{\lambda}\right)\left(\lambda\gamma^a\Pi\right). \tag{4.1}$$

As in the action of (2.25), the torsion term in (4.1) is not needed for covariance and was not included in [5], but simplifies the construction by removing the dependence of the b ghost on the Lorentz connection Ω_{Aab}.

To prove that $Qb = T$ where T is the stress-energy tensor of the heterotic string in a curved background, note that $S = S_0 + S_{\text{non-min}}$ of (2.15) and (2.25) implies that

$$T = -\frac{1}{2}\Pi_a\Pi^a - d_\alpha\Pi^\alpha - \omega_\alpha\nabla\lambda^\alpha + Q\left(-s^\alpha\nabla\widehat{\lambda}_\alpha + \frac{1}{4}\Pi^A T_{Aab}\left(s\gamma^{ab}\widehat{\lambda}\right)\right). \tag{4.2}$$

So one needs to show that

$$Qb_{\text{min}} = -\frac{1}{2}\Pi_a\Pi^a - d_\alpha\Pi^\alpha - \omega_\alpha\nabla\lambda^\alpha \tag{4.3}$$

where

$$b_{\text{min}} = -\omega_\alpha\Pi^\alpha + \Pi^a\overline{\Gamma}_a - \frac{1}{4\left(\lambda\widehat{\lambda}\right)}\left(\lambda\gamma^{ab}r\right)\overline{\Gamma}_a\overline{\Gamma}_b + \frac{1}{2\left(\lambda\widehat{\lambda}\right)}\left(\omega\gamma_a\widehat{\lambda}\right)\left(\lambda\gamma^a\Pi\right). \tag{4.4}$$

Although b_{min} is invariant under local Lorentz transformations, it transforms under the local scale transformation of (2.19) as

$$\delta b_{\text{min}} = -2\Lambda\Pi^a\overline{\Gamma}_a + 4\Lambda\frac{1}{4\left(\lambda\widehat{\lambda}\right)}\left(\lambda\gamma^{ab}r\right)\overline{\Gamma}_a\overline{\Gamma}_b \tag{4.5}$$

where we have used that $\overline{\Gamma}_a$ of (3.7) transforms as $\delta\overline{\Gamma}_a = -2\Lambda\overline{\Gamma}_a$. Using the definition of $\tilde{Q} = Q - Q_{L+S}$ in (3.5), (4.3) is therefore implied if

$$\tilde{Q}b_{\text{min}} + 2(\lambda\Omega)\Pi^a\overline{\Gamma}_a - \frac{(\lambda\Omega)}{\left(\lambda\widehat{\lambda}\right)}\left(\lambda\gamma^{ab}r\right)\overline{\Gamma}_a\overline{\Gamma}_b = -\frac{1}{2}\Pi_a\Pi^a - d_\alpha\Pi^\alpha - \omega_\alpha\nabla\lambda^\alpha. \tag{4.6}$$

Because of the similarity of (3.5) with the flat space BRST transformations of (2.10) and the result that $Q_{\text{flat}}b_{\text{flat}} = T_{\text{flat}}$, proving (4.6) only requires showing that the various factors of $(\lambda^\alpha\Omega_\alpha)$ coming from (3.5) and (3.8) cancel out in (4.6).

The first term in (4.4) contributes no factors of $(\lambda^\alpha\Omega_\alpha)$ and the second term in (4.4) contributes $-2(\lambda\Omega)\Pi^a\overline{\Gamma}_a$ from the $\tilde{Q}$ variation of $\overline{\Gamma}_a$. The third term in (4.4) contributes

$$\begin{aligned}\frac{1}{4(\lambda\widehat{\lambda})^2}&\left(5\left(\lambda\widehat{\lambda}\right)(\lambda\Omega)\right)\left(\lambda\gamma^{ab}r\right)\overline{\Gamma}_a\overline{\Gamma}_b - \frac{1}{4\left(\lambda\widehat{\lambda}\right)}\left(5(\lambda\Omega)\lambda^\alpha\right)\left(\gamma^{ab}r\right)_\alpha\overline{\Gamma}_a\overline{\Gamma}_b + \frac{(\lambda\Omega)}{\left(\lambda\widehat{\lambda}\right)}\left(\lambda\gamma^{ab}r\right)\overline{\Gamma}_a\overline{\Gamma}_b \\ &= \frac{(\lambda\Omega)}{(\lambda\widehat{\lambda})}\left(\lambda\gamma^{ab}r\right)\overline{\Gamma}_a\overline{\Gamma}_b\,,\end{aligned} \tag{4.7}$$

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where the first term comes from the variation of $(\lambda\widehat{\lambda})^{-1}$, the second term from the variation of λ^α, and the third term from the variation of $\overline{\Gamma}_a\overline{\Gamma}_b$. Finally, the fourth term in (4.4) contributes

$$-\frac{1}{2\left(\lambda\widehat{\lambda}\right)^2}\left(5(\lambda\Omega)\left(\lambda\widehat{\lambda}\right)\right)\left(\omega\gamma_a\widehat{\lambda}\right)(\lambda\gamma^a\Pi)+\frac{1}{2\left(\lambda\widehat{\lambda}\right)}3(\lambda\Omega)\left(\omega\gamma_a\widehat{\lambda}\right)(\lambda\gamma^a\Pi) \\ +\frac{1}{2\left(\lambda\widehat{\lambda}\right)}\left(\omega\gamma_a\widehat{\lambda}\right)5(\lambda\Omega)(\lambda\gamma^a\Pi)+\frac{1}{2\left(\lambda\widehat{\lambda}\right)}\left(\omega\gamma_a\widehat{\lambda}\right)(\lambda\gamma^a)_\alpha\frac{1}{4}\lambda^\beta T_{\beta bc}\left(\gamma^{bc}\Pi\right)^\alpha=0 \tag{4.8}$$

where the first term comes from the transformation of $(\lambda\widehat{\lambda})^{-1}$, the second term comes from the transformation of ω_α, the third term comes from the transformation of λ^α, and the last term comes from the transformation of Π^α. To show that (4.8) is zero, we have used that

$$\frac{1}{4\left(\lambda\widehat{\lambda}\right)}(\lambda\gamma_{bc}\Omega)(\lambda\gamma_a\gamma_{bc}\Pi)\left(\omega\gamma^a\widehat{\lambda}\right)=\frac{1}{4\left(\lambda\widehat{\lambda}\right)}(\lambda\gamma_{bc}\Omega)\left(\lambda\left([\gamma_a,\gamma_{bc}]+\gamma_{bc}\gamma_a\right)\Pi\right)\left(\omega\gamma^a\widehat{\lambda}\right) \\ =\frac{1}{\left(\lambda\widehat{\lambda}\right)}(\lambda\gamma_{ac}\Omega)(\lambda\gamma^c\Pi)\left(\omega\gamma^a\widehat{\lambda}\right)+\frac{1}{4\left(\lambda\widehat{\lambda}\right)}(\lambda\gamma_{bc}\Omega)(\lambda\gamma_{bc}\gamma_a\Pi)\left(\omega\gamma^a\widehat{\lambda}\right) \\ =-\frac{3}{2\left(\lambda\widehat{\lambda}\right)}(\lambda\Omega)(\lambda\gamma_a\Pi)\left(\omega\gamma^a\widehat{\lambda}\right). \tag{4.9}$$

So we have proven that the $(\lambda\Omega)$ factors cancel out in (4.6), and therefore $Qb=T$.

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A Computations in a flat background

In this appendix, we will prove the equation (2.14) for $Q\overline{\Gamma}_a$ and the equation $Qb=T$ in a flat background.

Using the BRST transformations of (2.10) acting on $\overline{\Gamma}_a$ of (2.12), one finds

$$Q\overline{\Gamma}_a=-\frac{1}{2\left(\lambda\widehat{\lambda}\right)}\Pi^b\left(\widehat{\lambda}\gamma_a\gamma_b\lambda\right)-\frac{1}{2\left(\lambda\widehat{\lambda}\right)^2}(\lambda r)\left(d\gamma_a\widehat{\lambda}\right)-\frac{1}{2\left(\lambda\widehat{\lambda}\right)}(d\gamma_a r) \\ -\frac{1}{4\left(\lambda\widehat{\lambda}\right)^3}(\lambda r)\left(r\gamma_{abc}\widehat{\lambda}\right)N^{bc}-\frac{1}{8\left(\lambda\widehat{\lambda}\right)^2}(r\gamma_{abc}r)N^{bc}+\frac{1}{16\left(\lambda\widehat{\lambda}\right)}\left(r\gamma_{abc}\widehat{\lambda}\right)\left(\lambda\gamma^{bc}d\right). \tag{A.1}$$

The term independent of r agrees in (2.14) and (A.1), and the term linear in r in (A.1) is

$$-\frac{1}{2\left(\lambda\widehat{\lambda}\right)^2}(\lambda r)\left(d\gamma_a\widehat{\lambda}\right)-\frac{1}{2\left(\lambda\widehat{\lambda}\right)}(d\gamma_a r)+\frac{1}{16\left(\lambda\widehat{\lambda}\right)^2}\left(r\gamma_a\gamma_{bc}\widehat{\lambda}\right)\left(\lambda\gamma^{bc}d\right) \\ =-\frac{1}{2\left(\lambda\widehat{\lambda}\right)^2}(\lambda r)\left(d\gamma_a\widehat{\lambda}\right)+\frac{1}{4\left(\lambda\widehat{\lambda}\right)^2}(\lambda\gamma_b\gamma_a r)\left(d\gamma^b\widehat{\lambda}\right) \\ =-\frac{1}{4\left(\lambda\widehat{\lambda}\right)^2}(\lambda\gamma_a\gamma_b r)\left(d\gamma^b\widehat{\lambda}\right)=\frac{1}{8\left(\lambda\widehat{\lambda}\right)^3}(\lambda\gamma_{bc}r)\left(\widehat{\lambda}\gamma^c\gamma_a\lambda\right)\left(d\gamma^b\widehat{\lambda}\right) \tag{A.2}$$

which is the term linear in r in (2.14). To go from the first line to the second line of (A.2), we have used the identity

$$\frac{1}{2}\delta_\alpha^\delta\delta_\beta^\gamma+\frac{1}{16}\left(\gamma^{bc}\right)_\alpha{}^\gamma(\gamma_{bc})_\beta{}^\delta=\frac{1}{4}\gamma^b_{\alpha\beta}\gamma_b^{\gamma\delta}-\frac{1}{8}\delta_\alpha^\gamma\delta_\beta^\delta \tag{A.3}$$

together with the pure spinor constraints of (2.1).

Finally, the terms quadratic in r in (A.1) are

$$\begin{aligned}
&-\frac{1}{8\left(\lambda\widehat{\lambda}\right)^2}(r\gamma_{abc}r)N^{bc}-\frac{1}{4\left(\lambda\widehat{\lambda}\right)^3}(\lambda r)\left(r\gamma_{abc}\widehat{\lambda}\right)N^{bc} \\
&=-\frac{1}{8\left(\lambda\widehat{\lambda}\right)^2}(r\gamma_{abc}r)N^{bc}-\frac{1}{384\left(\lambda\widehat{\lambda}\right)^3}\left(\lambda\gamma_{def}\gamma_{bc}\gamma_a\widehat{\lambda}\right)\left(r\gamma^{def}r\right)N^{bc} \\
&=-\frac{1}{8\left(\lambda\widehat{\lambda}\right)^2}(r\gamma_{abc}r)N^{bc}+\frac{1}{16\left(\lambda\widehat{\lambda}\right)^3}\left(\lambda\gamma_b\gamma_a\widehat{\lambda}\right)\left(r\gamma^{bde}r\right)N_{de} \\
&=-\frac{1}{16\left(\lambda\widehat{\lambda}\right)^3}(\lambda\gamma_a\gamma_b r)\left(r\gamma^{bde}\widehat{\lambda}\right)N_{de},
\end{aligned} \tag{A.4}$$

which is the term quadratic in r in (2.14). To go from the first line to the second line of (A.4), we have used the identity $r_\alpha r_\beta=\frac{1}{96}\gamma^{def}_{\alpha\beta}\left(r\gamma_{def}r\right)$. To go from the second line to the third line, we have used that $\left(\lambda\gamma^b\right)_\alpha N_{bc}=\frac{1}{2}J(\gamma^c\lambda)_\alpha$ where $J=-\lambda^\alpha\omega_\alpha$ and that all terms proportional to J vanish using the pure spinor constraints of (2.1). And to go from the third line to the fourth line, we have used that $(\gamma_b r)^\alpha\left(\gamma^b\widehat{\lambda}\right)^\beta=-\left(\gamma_b\widehat{\lambda}\right)^\alpha\left(\gamma^b r\right)^\beta$. So we have proven that $\overline{\Gamma}_a$ satisfies equation (2.14) in a flat background.

We now verify that $Qb=T$ in flat space. Applying Q of (2.10) to (2.13), we obtain

$$\begin{aligned}
Qb=T+(\lambda\gamma^a\Pi)&\left(\frac{1}{8\left(\lambda\widehat{\lambda}\right)^2}\left(r\gamma_{abc}\widehat{\lambda}\right)N^{bc}-\frac{1}{2\left(\lambda\widehat{\lambda}\right)^2}(\lambda r)\left(\omega\gamma_a\widehat{\lambda}\right)+\frac{1}{2\left(\lambda\widehat{\lambda}\right)}(\omega\gamma_a r)\right) \\
&-\frac{1}{4\left(\lambda\widehat{\lambda}\right)^2}\overline{\Gamma}^a\overline{\Gamma}^b\left((\lambda r)(\lambda\gamma_{ab}r)+\frac{1}{2\left(\lambda\widehat{\lambda}\right)}\left(\lambda\gamma_a{}^c r\right)(\lambda\gamma_{bd}r)\left(\widehat{\lambda}\gamma^d\gamma_c\lambda\right)\right).
\end{aligned} \tag{A.5}$$

Using the identity

$$\left(\lambda\widehat{\lambda}\right)(\omega\gamma_a r)-(\lambda r)\left(\omega\gamma_a\widehat{\lambda}\right)=-\frac{1}{8}\left(\widehat{\lambda}\gamma_{abc}r\right)N^{bc}-\frac{1}{48}(\lambda\gamma_{abcd}\omega)\left(r\gamma^{bcd}\widehat{\lambda}\right), \tag{A.6}$$

we obtain that the second term in (A.5) is equal to

$$\frac{1}{16\left(\lambda\widehat{\lambda}\right)}(\lambda\gamma^a\Pi)\left(\left(r\gamma_{abc}\widehat{\lambda}\right)N^{bc}+\frac{1}{6}(\lambda\gamma_{abcd}\omega)\left(r\gamma^{bcd}\widehat{\lambda}\right)\right), \tag{A.7}$$

which can be seen to vanish using $(\lambda\gamma^a)_\alpha(\lambda\gamma_a)_\beta=0$. Finally, the term proportional to $\overline{\Gamma}^a\overline{\Gamma}^b$ in (A.5) vanishes using the identities $(\lambda\gamma^a)_\alpha(\lambda\gamma_a)_\beta=(\lambda r)(\lambda r)=0$.

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