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Magnetic field seed generation in plasmas around charged and rotating black holes

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Abstract

Previous work by the authors introduced the possibility of generating seed magnetic fields by spacetime curvature and applied it in the vicinity of a Schwarzschild black hole. It was pointed out that it would be worthwhile to consider the effect in other background geometries and particularly in the vicinity of a rotating black hole, which is generically to be expected, astrophysically. In this paper that suggestion is followed up and we calculate generated magnetic field seed due to Reissner–Nördstrom and Kerr spacetimes. The conditions for the drive for the seed of a magnetic field is obtained for charged black holes, finding that in the horizon the drive vanishes. Also, the ψN-force produced by the Kerr black hole is obtained and its relation with the magnetic field seed is discussed, producing a more effective drive.

Keywords: seed magnetic field, general relativistic drive, Kerr metric

(Some figures may appear in colour only in the online journal)

1. Introduction

Most astronomical objects have magnetic fields. If one thinks about it, this fact is surprising. The early universe was very homogenous and isotropic, as is evidenced by the microwave background. Magnetic fields, however, have preferred directions. How did such fields originally develop? That they have been significant in the universe since fairly early times is borne out by the extremely high energies achieved by cosmic rays, that seem to have been provided by very high magnetic fields. The problem of generating such magnetic fields has been addressed in the literature [1–3]. It is to be expected that only a small field would have been generated spontaneously initially and that field would later have been enhanced by some nonlinear processes later.

Mahajan and Yoshida [4] suggested, that the fields could be generated by a sufficiently hot plasma due to special relativistic effects. Later its implications were studied [5]. It relies on the length contraction involved in rotational motion [6] that had been pointed out by Einstein himself [7] and led him to the conclusion that the geometry of the space could not be Euclidean. The reason is that there would be length contraction along the direction of motion but not perpendicular. Thus the rim would be reduced in circumference but the diameter would be unaltered and hence their ratio would not be π. It is natural, then, to ask whether the seed magnetic field generation could not be caused by *spacetime curvature*. This was proposed by the present authors [8]. We used the Schwarzschild black hole as the background to provide the curvature for explicitly estimating the magnitude of the effect. It would be more realistic to consider the Kerr (spinning) black hole to provide the effect. In this paper the previous calculations are extended to the charged and rotating black holes and the modification of the effects obtained.

The extension to general relativity can be obtained by other means but it can also be understood in terms of a formalism that re-expresses general relativity in terms of a modified gravitational force [9, 10]. This is convenient for the further extensions pursued in this paper. The plan of the paper is as follows. In the next section a brief review of this formalism is provided and its application to the general relativistic drive is provided. In the third section a brief review of the proposal for magnetic seed generation by a

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special relativistic drive for plasmas is given. In the subsequent section the formalism is applied to the extension of the earlier proposal to a plasma around a Reissner–Nördstrom (charged) black hole, for which the proposal can be directly extended. For the more realistic Kerr (rotating) black hole the proposal needs to be slightly re-worked. This is done in section five. A brief summary and discussion follows in the last section.

2. Review of pseudo-Newtonian formalism

The 'pseudo-Newtonian', or ψN-formalism, takes a preferred frame corresponding to an observer falling freely in a gravitational field either from rest at infinity, or at any place with a velocity corresponding to that which would have been achieved if it had fallen freely at rest from infinity to that place [9, 10]. While relativity replaces forces by the curvature of a spacetime, this formalism effectively 'straightens out' the spacetime and writes the geodesics *as if* they were straight lines bent due to an (appropriately modified) force of gravity called a ψN-force. The observer then sees a Minkowski space and so the corresponding spacelike sections are flat. Hence the frame is given by a flat foliation of the spacetime [11, 12]. It is a special Fermi–Walker frame [13].

The basic idea is that whereas a freely falling observer does not feel the force of gravity, the *tidal acceleration* can still be measured, as the geodesic deviation

$$\mathcal{A}^{\mu} = R^{\mu}_{\nu\rho\pi} t^{\nu} p^{\rho} t^{\pi}, \tag{1}$$

where $R^{\mu}_{\nu\rho\pi}$ is the curvature tensor, t^{ν} is the unit timelike vector along the geodesic giving the observer's world line and p^{ρ} is the position vector of a point on a neighbouring geodesic seen by the observer. This can be modeled by an 'accelerometer' consisting of a cylinder containing two equal masses connected by a spring. The p^{ρ} now represents the accelerometer. The observer holds one end and the spring ends in a needle attached to the other end. The needle swivels freely on a dial so that as the spring is stretched the needle shifts to the positive side, giving a measure of the stretching force and to the negative side if the spring is compressed, giving a measure of the compressing force. By turning the accelerometer till the maximum reading is obtained, a freely falling observer in a closed laboratory would be able to measure the direction and magnitude of the stretching or squashing tidal force on a unit test mass.

The tidal force in normal Newtonian gravity would have a similar effect on the accelerometer. It is the directional derivative of the Newtonian gravitational force. Since we have effectively replaced the curved spacetime of relativity by Minkowski spacetime with a force bending the geodesics, we can define the ψN-force to be that quantity whose directional derivative along the accelerometer is the tidal force. This works properly when we take the maximized magnitude of the tidal force. Since normal gravity causes stretching, one gets a measure of the gravitational force along the direction the accelerometer is held. If there were a repulsive gravity component or the force of gravity acted less, the stretching would be reduced. This would apply in a situation even where the repulsive component dominates as one could then maximize the negative reading. The single accelerometer could be upgraded to a cluster and the gravitational force could then be fully mapped by the observer.

Here $\mathbf{t}$ would be taken to provide the basis vector for time and hence provide a frame. The preferred frame is the one corresponding to the observer falling freely from rest at infinity. The gravitational force so defined depends on the full zero-zero component of the metric tensor and hence gives the relativistic generalization of the Newtonian force. One could ask for the potential that gives this force. For static or stationary spacetimes one can easily then define the potential that would yield this force, which we call the ψN-potential. This turns out to be [14] $\ln \sqrt{g_{oo}}$. Of course, in general one would need to use the full metric tensor for all components of the potential to play a role. Nevertheless, the gauge freedom would allow reduction to six potentials in a preferred frame. It is the time invariance of the metrics chosen that makes it so convenient to construct the preferred frame, even if we have only circular symmetry. (For completeness it may be mentioned that the formalism is also extended to non-stationary spacetimes and yields the momentum imparted to test particles by a time varying gravitational field as well [15].)

3. Review of Proposal

The essential problem is to find a way of getting a preferred direction, of the magnetic field, when there was none to start with. Fluctuations in a plasma would lead to transient magnetic fields that come and go, but we need a mechanism to get a magnetic field that sustains itself long enough for nonlinear effects to make it grow, so that small fluctuations cannot wipe it out.

There is a theorem that no fields will develop in a plasma spontaneously. This comes from the fact that the circulation $\oint_{L(t)} \delta Q$, associated with a physical quantity δQ, calculated along the moving loop $L(t)$, may be zero or finite depending on whether $\oint_{L(t)} \delta Q$ is an exact differential $d\varphi$ (φ being a state variable). Due to conservation of momentum, the rate of change of the circulation associated with the canonical momentum would be zero. This would amount to the closed integral of the differential of the total energy of the system over the loop, which must be zero. Here the energy would be the sum of the kinetic and potential energies *and the enthalpy*.

For a rotating plasma, as mentioned earlier, there is an extra inverse of the Lorentz factor to be associated with the loop. Thus, what was an exact differential does not remain an exact differential.

The canonical momentum need not necessarily only be the usual momentum but can also include the angular momentum. In the absence of any intrinsic angular momentum,

$$M^{\mu\nu} = \frac{q}{c}F^{\mu\nu} + 2mc\partial^{[\mu}\left(fU^{\nu]}\right), \tag{2}$$

where $F^{\mu\nu}$ is a generalized Maxwell tensor for any generalized 4-vector potential A^{μ}, the four-velocity of the plasma is U^{μ}, c is the speed of light, q and m is the charge and mass, the average factor of increase of mass due to thermal motion is represented by the f function, and the square bracket stands for the skew of a tensor quantity, which incorporates a half. For non-relativistic temperatures T (i.e. when the speeds of the particles are much less than the speed of light) $f \approx 1$ but would be significantly greater than 1 at relativistic temperatures. For example, $f \approx 20/3$ for $T = 1\ MeV$.

The generalized force, incorporating thermodynamic effects, is:

$$\mathbf{F}_{\mathbf{L}} = \frac{cT}{\gamma}\nabla\sigma, \tag{3}$$

where σ is the entropy, and γ is the Lorentz factor. If the curl of this force were zero no field would be generated as the force would be irrotational. However, if there is a temperature gradient or a varying Lorentz factor, a generalized magnetic field would be generated. The former yields the usual baroclinic field

$$\widetilde{\mathbf{B}}_{\mathbf{b}} = -\frac{c}{q\gamma}\nabla T \times \nabla\sigma, \tag{4}$$

and the latter yields the special relativistic drive

$$\widetilde{\mathbf{B}}_{\mathbf{r}} = -\frac{c\gamma T}{2q}\nabla\left(\frac{v}{c}\right)^2 \times \nabla\sigma, \tag{5}$$

where v is the velocity.

If the kinematic and temperature gradients have the same magnitude, the ratio of the relativistic to the baroclinic fields is $(\gamma v/c)^2$, which can become extremely large. We need that this drive, to generate the generalized magnetic field, should be greater than the resistive dissipative term, so that magnetic fields can build up. The ratio of the relativistic drive to the dissipation, under some reasonable approximations, is

$$R_r \approx \frac{(\gamma v/c)^2\left(T/mc^2\right)}{(v_A/c)(\nu/\omega_p)}, \tag{6}$$

where v_A is the Alfven speed, ω_p is the plasma frequency and ν is the collision frequency. For sufficiently large rotational and thermal velocities this ratio can be very significant in the early stages. For an electron gas of density 10^{10}/cc at $T \sim 20$ eV and $v/c \sim 10^{-2}$, the relativistic drive dominates over the dissipation till a magnetic field of $\sim$1 G is reached, after which the dissipation would take over.

One might wonder how a change of frame of reference can make a physical difference. Any change made can be undone. However, for one thing we have a rotating plasma that provides the vorticity (if it is not isothermal and isentropic) and for another, we are strictly speaking not using special relativity, which applies only for constant velocities, but have a changing velocity, despite having a constant speed. (For completeness it should be mentioned that the original ‘restricted’ theory of relativity applied for only inertial frames and not for accelerated frames and the general theory removed that restriction. Gravity was originally used as a *tool* for removing the restriction. The general theory was not originally formulated as a theory of gravity.) Of course, we have ‘smuggled in’ the preferred direction with the rotating plasma. As such, it is not so surprising that we have been able to generate a magnetic field from it. Can one do better?

The authors of this paper proposed a general relativistic analogue of this drive [8]. The point is that we can take the local rest-frame at one point of a curved spacetime, as given by the tangent space using Riemann normal coordinates, and compare it with the local rest-frame at another point. There will be a definable local Lorentz factor there, giving the special relativistic effect, now produced by gravity. The GR effects open up the exciting possibility of spontaneous generation of magnetic fields near gravitating sources without appealing to relativistic rotational speeds of plasmas associated with relativistic temperatures.

Now, use the canonical break-up of the metric tensor,

$$ds^2 = \alpha^2 dt^2 - 2\beta_i dt dx^i - \Gamma_{ij}dx^i dx^j, \tag{7}$$

where α is called the lapse function, β_i the shift vector and Γ_{ij} is the 3-space metric tensor. For the ψN-frame the shift vector is zero and the potential is ln α. Clearly, one can extend the 3-metric to 4-dimensions in the canonical formalism by defining $\Gamma_{\mu\nu} = t_\mu t_\nu - g_{\mu\nu}$. The local Lorentz factor is $\gamma = \left(\alpha^2 - \Gamma_{\mu\nu}t^\mu t^\nu\right)^{-1/2}$. One again gets a generalized magnetic field $\mathbf{B}$, a generalized electric field and thus a generalized vorticity vector

$$\Omega = \mathbf{B} + \frac{mc}{q}\nabla \times \left(f\gamma\mathbf{v}\right). \tag{8}$$

The generalized vorticity evolves due to effect of the baroclinic and the (general) relativistic terms:

$$\Omega_{,t} - \nabla \times \left(\mathbf{v} \times \Omega\right) = \widetilde{\mathbf{B}}_{\mathbf{b}} + \widetilde{\mathbf{B}}_{\mathbf{r}}. \tag{9}$$

The baroclinic term stays as it was but the new relativistic drive now becomes

$$\widetilde{\mathbf{B}}_{\mathbf{r}} = \frac{c\gamma T}{2q}\nabla\left[\left(\frac{v}{c}\right)^2 - \alpha^2\right] \times \nabla\sigma. \tag{10}$$

The ratio of the two terms here is

$$\left|\frac{B_r}{B_b}\right| \approx \frac{\gamma^2\tau}{2}\left|\nabla\left[\left(\frac{v}{c}\right)^2 - \alpha^2\right]\right|, \tag{11}$$

where τ is the scale length of variation of temperature, rescaled to incorporate the factor due to curvature, coming from the square-root of the lapse function, α. This factor can, in principle, be arbitrarily large if the two gradients are comparable in magnitude and have different directions. Notice that the second gradient is related to the ψN-force.

The estimates were used for simplicity for an electron-positron plasma around a Schwarzschild black hole:

$$ds^2 = e^{\nu(r)}dt^2 - e^{-\nu(r)}dr^2 - r^2\left(d\theta^2 + \sin^2\theta d\phi^2\right), \quad (12)$$

where $e^{\nu(r)} = 1 - r_o/r$ and $r_o = 2Gm\, c^{-2}$ is the Schwarzschild radius, m being the black hole mass. The baroclinic term will be zero as the entropy will be just a function of temperature of the accretion disc, $\sigma = f\left(T\right)$, and so the gradients of the two will be parallel. Taking the usual definition of entropy, which is valid beyond about $5r_o$ as significant nonlinearity effects come in only closer, the relativistic drive comes out to be

$$\left|\widetilde{\mathbf{B}}_{\mathbf{r}}\right| = \frac{3\zeta\sigma' r_o e^{\nu/2}}{4qr^3}\left(1 - \frac{3r_0}{2r}\right)^{-1/2} T_\phi, \quad (13)$$

where q is now the electron charge, ζ is of order unity, $\sigma' = \partial_T\sigma$, and $T_\phi = \partial_\phi T$. Taking the plasma to be in a thin disc the drive will act along the z-axis and the plasma will lie in the xy plane. For $5r_o$ this gives a very small drive (for a solar mass black hole) at the temperature to be expected [16], $\sim 10^{-6}$ G. However, as we go closer in, the seed field rises exponentially. The calculation would become more tricky in the details due to the modification required for the definition of entropy, but the basic estimates would largely carry through.

4. Extension to Reissner-Nördstrom and Kerr black holes

We will first consider the electrically charged, or Reissner–Nördstrom, black hole. Since astrophysical bodies are assumed to be charge neutral, it is not expected to have any physical relevance. Nevertheless, it is instructive to try to extend the above calculations to it, since it has a metric, which is the same as the Schwarzschild but with $e^{\nu(r)} = 1 - 2GM/c^2r + GQ^2/c^4r^2$, Q being the charge in appropriate units. We use the ψN formalism for the extension rather than a more precise calculation with back-reactions taken into account, as we are interested in an estimate of the effect that gives physical insights rather than in its precise magnitude. Now, here

$$\left|\nabla e^\nu\right| = 2G\left|\frac{M}{r^2c^2} - \frac{Q^2}{c^4r^3}\right|. \quad (14)$$

Since the difference from the Schwarzschild comes from the second term, which is subtracted from the first, there is a *reduction* of the drive a by Mrc^2/Q^2. The black hole radius is now

$$r_+ = \frac{GM}{c^2}\left(1 + \sqrt{1 - \frac{Q^2}{GM^2}}\right), \quad (15)$$

which will be real only if $Q^2 \leqslant GM^2$. For the equality it is called an *extreme* black hole and the radius becomes half the Schwarzschild value. In this case the drive reduces to $\frac{2G^2M^2}{c^4r^2}\left(r/r_+ - 1\right)$. Thus, at the minimum value of r, namely r_+, this reduction *will completely eliminate the drive*.

The more realistic modification is for a spinning, or Kerr, black hole, with angular momentum per unit mass a. Here another complication arises. The point is that there is frame-dragging due to the Lense–Thirring effect [13], which changes with the polar angle, θ, being maximum at the equator and zero at the poles. This leads to an off-diagonal ($t - \phi$) term in the metric and gives non-trivial dependence on θ, apart from r. The metric now becomes,

$$ds^2 = Ac^2dt^2 - Bdr^2 - \rho^2d\theta^2 - Cd\phi^2 + 2Dcdtd\phi, \quad (16)$$

where, putting $R = Gm/c^2$ and using gravitational units $c = G = 1$

$$\begin{aligned}
A &= \frac{\chi^2}{\rho^2}\,, \\
B &= \frac{\rho^2}{\Delta}\,, \\
\rho^2 &= r^2 + a^2\cos^2\theta\,, \\
\chi^2 &= r^2 - 2Rr + a^2\cos^2\theta\,, \\
\Delta &= r^2 - 2Rr + a^2\,, \\
C &= \left(r^2 + a^2 + \frac{2Rra^2}{\rho^2}\sin^2\theta\right)\sin^2\theta\,, \\
D &= \frac{2Rra}{\rho^2}\sin^2\theta\,,
\end{aligned} \quad (17)$$

and the black hole radius is at

$$r_+ = R + \sqrt{R^2 - a^2}. \quad (18)$$

The ψN-potential is $\phi = \ln\sqrt{g_{oo}}$ and the corresponding ψN-force is

$$\nabla\phi = \frac{R}{\rho^2\chi^2}\left(r^2 - \frac{a^2}{c^2}\cos^2\theta, -2r\frac{a^2}{c^2}\sin\theta\cos\theta, 0\right). \quad (19)$$

The radial component is decreased but there is an additional component to be accounted for. To obtain the magnitude of this vector we must remember to use the metric tensor components. As this vector has only radial and polar components and the metric tensor is diagonal in these, we need only multiply each squared component with the corresponding metric component and add the two to get the squared magnitude of the ψN-force. The general relativistic contribution to the strength (and direction) of the drive (for the magnetic field seed) is measured by the gradient of the ψN-potential while the total strength of the drive is calculated from its vector product with the entropy gradient [see equation (10)]. Since the ψN-force as computed here is given by the gradient of the ψN-potential, we need to use the corresponding components of the *contravariant* Kerr metric tensor. The squared magnitude is then [17]

$$F_{\psi N}^2 = \frac{R^2}{\rho^6\chi^4}\Big[r^6 - 2Rr^5 - a^2r^4\cos 2\theta + 4Ra^2r^3\cos^2\theta + \\ + a^4r^2\cos^2\theta\left(2 - 3\cos^2\theta\right) \\ - 2Rra^4\cos^4\theta + a^6\cos^4\theta\Big]. \tag{20}$$

This is a complicated expression and there is no apparent unique value of θ for which it is maximum, as the value will be r-dependent. As such, it is worth considering the special cases $\theta = 0, \pi, \pi/2$.

At the poles ($\theta = 0$ or π)

$$F_{\psi N}^2 = \frac{R^2\left(r^2 - a^2\right)^2}{\left(r^2 + a^2\right)^3\left(r^2 - 2Rr + a^2\right)}. \tag{21}$$

It is obvious that there is a *decrease* of the ψN-force here compared with the Schwarzschild black hole value $F_{\psi N}^2 = R^2/\left(r^3\left(r - 2R\right)\right)$ at the corresponding distance as the numerator is decreased and the denominator increased and the reduction is more for larger a. Whereas, at the surface of the Schwarzschild black hole this would become infinite, for the Kerr metric it would tend to $1/2R^2$.

At the equator ($\theta = \pi/2$) the ψN-force reduces to

$$F_{\psi N}^2 = \frac{R^2\left(r^2 - 2Rr + a^2\right)}{r^4\left(r - 2R\right)^2}. \tag{22}$$

It is to be noted that this term blows up at $r = 2R$, which lies in the accessible region $r > r_+$, while it would come on the horizon for the Schwarzschild metric. As the spin is increased, the distance from the singular place increases, going to a maximum of twice the horizon in the extreme case. Of course, this entire region is inaccessible to comparison with the Schwarzschild drive, as it lies beyond the accessible range for it. Hence the drive is much stronger outside the horizon for the Kerr drive on the equator, than for the Schwarzschild drive. On the other hand, the numerator contains the term $(r - r_+)^2$, which will become zero at the horizon. Hence the force disappears there. At the extreme case, the force goes as $R^2\left(r - R\right)^2/\left[r^4\left(r - 2R\right)\right]$. Going through the analysis in this case, it can again be seen to be zero at the horizon.

We can then conclude that on the surface of the hole this force is zero, which is consistent with the fact that the Hawking temperature of an extreme black hole is zero. Does this mean that our drive is negligible for the Kerr black hole? On the contrary, for a Kerr black hole the term $\left(r - 2R\right)^2$ in the denominator makes it much more effective than for the Schwarzschild black hole.

To visualize the ψN-force magnitude more easily we have plotted it for four different values of $\beta = a/Rc$, namely 0, 0.3, 0.7 and 1.0. These are shown in figures 1 (a)–(d). For a fixed distance, the maximum of the force appears near of $\pi/2$. The variation of the force with a is generally displayed for specific values of θ in figure 2.

5. Summary and discussion

A special relativistic explanation for magnetic field seed generation in a plasma [4, 5] had been followed up by us with a general relativistic explanation [8]. There it had been suggested that the proposal should be extended to the rotating black hole. This suggestion has been pursued here. It was noted that while the seed generation would be reduced for a charged black hole, totally disappearing for the extreme case, this would not apply for a rotating black hole. One might wonder why this should be so. The answer is that the charge acts in the opposite way to the mass. This fact had been noted by Maxwell himself, who constructed complicated mechanical models of the aether to explain it [18]. It comes out sharply in relativity, where charges give a *repulsive* effect on neutral matter [19]. The spin, on the other hand, causes only frame dragging and hence does not decrease the gravitational force.

We have qualitatively argued that no substantial reduction is to be expected for the rotating black hole, and given an estimate for the effect at various values of the parameters. Notice that the frame dragging changes a straight line into a conical spiral and hence mixes the radial dependence with a polar angle dependence. The relevant distance for Schwarzschild is measured in terms of $2R$, but for Kerr is r_+, which could be as low R. The relativistic drive can be calculated from the vector product of the gradient of the ψN -potential (19) and the gradient of the entropy of the plasma. Its exact value will depend on the thermal properties of the plasma, but its existence is due exclusively to the ψN -potential (19) or the ψN-force (20). Thus, comparing the drive with the same effect in a Schwarzschild spacetime, the ratio of the two drives at the appropriate distances is the relevant quantity. Consequently, the Kerr drive would work slightly better than the Schwarzschild drive. As had been stressed in our earlier paper, closer to the black hole the drive gets enhanced. Light goes into circular orbit around a Schwarzschild black hole at only 1.5 times the horizon distance. Outside that we could still get orbits that would not necessarily fall in very quickly. For the Kerr black hole the orbits are more complicated with elliptical paths that can come closer in, involving polar angle changes as well. However, that relatively stabilizes the orbits, as they tend to oscillate northward and southward as well, instead of only going inward [20]. This effectively stabilizes the orbit for a bit longer.

It is of particular relevance to remark that the plasma in the accretion disc is likely to lie on (or near) the equator and not at an oblique angle to the axis of rotation. As such, though the Kerr drive totalled over the entire solid angle may be only slightly larger than for the Schwarzschild, the plasma about a Kerr black hole receives a stronger ‘kick’ than it would around the Schwarzschild black hole. The fact that our

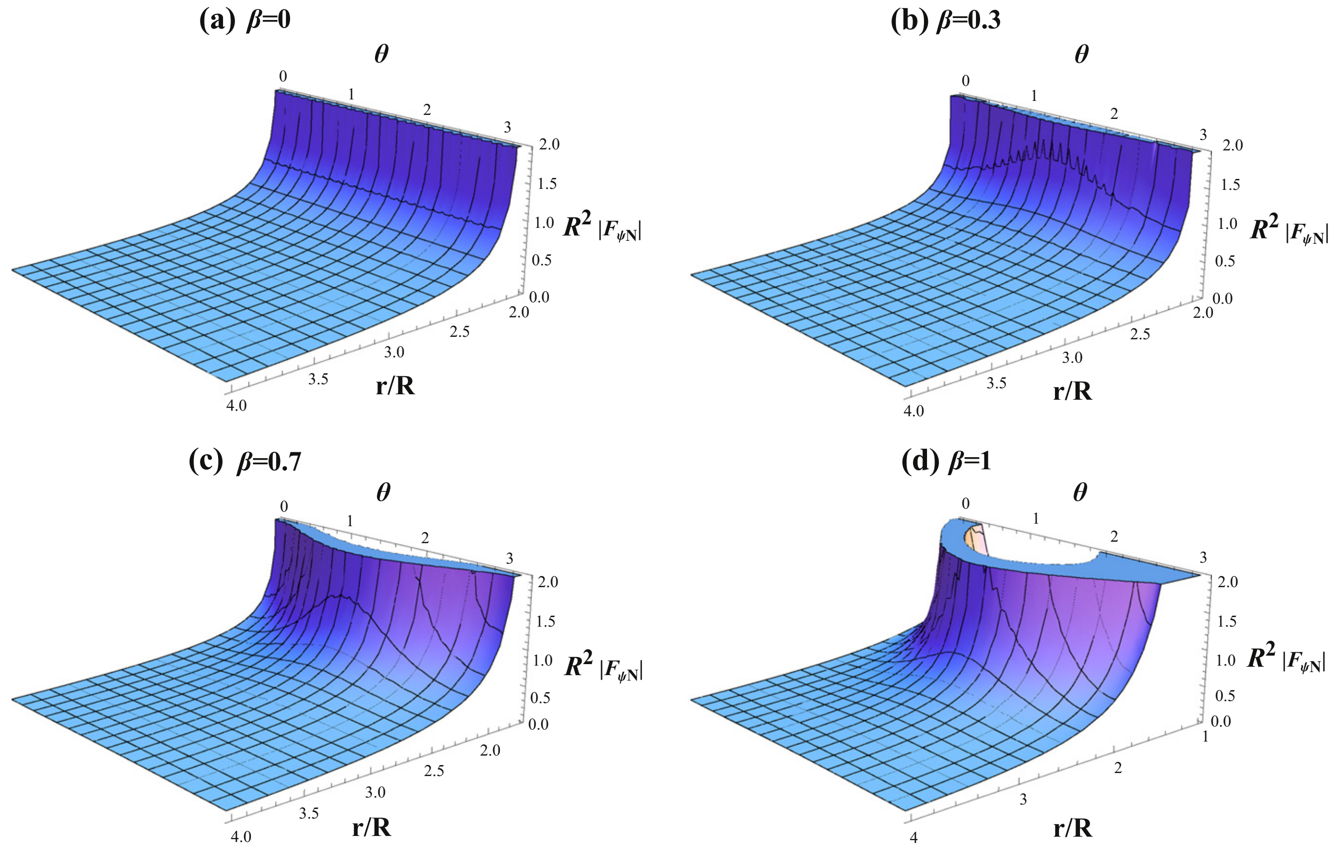

Figure 1. The normalized magnitude of the ψN-force (20) is plotted in terms of the normalized distance r/R and θ, for different values of the normalized angular momentum per unit mass $\beta = a/R$. (a) The Schwarzschild case with $\beta = 0$. (b) $\beta = 0.3$. (c) $\beta = 0.7$. (d) The extreme case of $\beta = 1$.

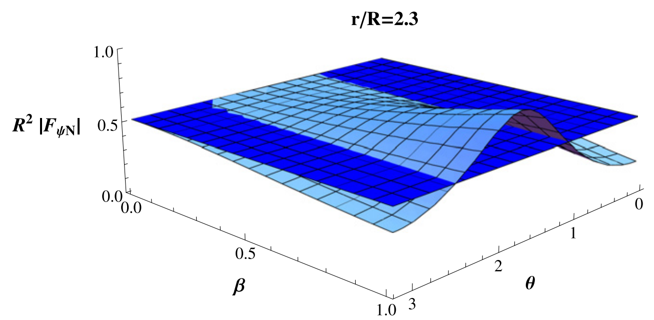

Figure 2. The normalized magnitude of the ψN-force (20) for a given distance $r/R = 2.3$ in terms of θ and $\beta = a/(Rc)$. The dark blue plane is the Schwarzschild drive and the light blue curve is the Kerr drive. The Kerr drive could be smaller or greater than the Schwarzschild drive depending of the value of $\beta = a/R$. For $a \neq 0$, the Kerr drive in the poles is always smaller than the Schwarzschild drive, while on the equator the Kerr drive is always greater than the Schwarzschild drive. Notice how the maximum of the Kerr drive is around $\pi/2$.

approximation is limited makes it difficult to estimate the extent of enhancement.

For completeness it is worth discussing, also, the force for the charged Kerr black hole. The only difference here is that g_{oo} has a GQ^2/c^4 subtracted inside the bracket and Δ has it added. The black hole radius is now at

$$r_+ = \frac{GM}{c^2}\left(1 + \sqrt{1 - \frac{a^2c^2 + GQ^2}{G^2 M^2}}\right), \tag{23}$$

The extreme case arises when the discriminant is zero. As such, the radius is again just the mass of the extreme black hole in gravitational units. The calculation follows exactly the same procedure as for the uncharged case. Since the Δ does not enter into the ψN-potential, we only have to account for the change in the g_{oo} for the gradient and then incorporate the modification of Δ for the squared magnitude. The result is an addition of

$$\frac{GQ^2}{c^4}\left(r^2 - \frac{a^2}{c^2}\cos^2\theta\right)^2 \tag{24}$$

to the expression in the square brackets in equation (20).

The more precise calculation with the back-reaction included would be worth obtaining, despite the fact that we are using broad guesses in any case, as we do not know the precise nature of the plasma or the black hole being investigated. The point is that it could, in principle, substantially diminish or enhance the drive, despite the intuition based on a linear theory that the difference would not be much. Still better would be simulations based on numerical relativity calculations for the field generated in the plasma of an accretion disc by a fast spinning black hole for different mass black holes. We leave that work to experts in the field of numerical relativity.

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References

[1] Kulsrud R M and Zweibel E G 2008 *Rep. Prog. Phys.* **71** 046901
[2] Midrow L M 2002 *Phys. Mod. Rev.* **74** 775
[3] Parker E N 1979 *Cosmical Magnetic Fields: Their Origin and Their Activity* (New York: Oxford University Press)
[4] Mahajan S M and Yoshida Z 2010 *Phys. Rev. Lett.* **105** 095005
[5] Mahajan S M and Yoshida Z 2011 *Phys. Plasmas* **18** 055701
[6] Qadir A 1989 *Relativity: An Introduction to the Special Theory* (Singapore: World Scientific)
[7] Einstein A 1950 *The Meaning of Relativity* 3rd ed (Princeton, NJ: Princeton University Press)
[8] Asenjo F A, Mahajan S M and Qadir A 2013 *Phys. Plasmas* **20** 22901
[9] Mahajan S M, Qadir A and Valanju P M 1981 *Nuovo Cimento* B **65** 404
[10] Qadir A 1997 *Nuovo Cimento* B **112** 485
[11] Hussain V, Qadir A and Siddiqui A A 2002 *Phys. Rev.* D **65** 027501
[12] Qadir A and Siddiqui A A 2002 *Nuovo Cimento* B **117** 909
Qadir A and Siddiqui A A 2006 *Int. J. Mod. Phys.* D **15** 1419
[13] Misner C W, Thorne K S and Wheeler J A 1973 *Gravitation* (San Fransico, CA: WH Freeman)
[14] Qadir A and Quamar J 1986 *Europhys. Lett.* **2** 422
[15] Qadir A and Sharif M 1992 *Nuovo Cimento* B **107** 1071
Qadir A and Sharif M 1992 *Phys. Lett.* A **167** 331
[16] Vietri M 2008 *Foundations of High-energy Astrophysics* (Chicago: The University of Chicago Press)
[17] Qadir A 1986 *Europhys. Lett.* **2** 427
[18] Maxwell J C 1865 *Phil. Trans. Royal Soc. (London)* **155** 459
[19] Qadir A 1983 *Phys. Lett.* A **99** 419
[20] Chandrasekhar S 1998 *The Mathematical Theory of Black Holes* (Oxford: Oxford University Press)