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Correcting the Market Failure in Work Trips with Work Rescheduling: An Analysis Using Bi-level Models for the Firm-workers Interplay

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Abstract Compulsory trips (e.g., work trips) contribute with the major part of the congestion in the morning peak. It also prevents the society to reach a social optimum (the solution that maximizes welfare) because the presence of the private utility of one the agents (the firm), acting as a dominant agent, does not account for the additional costs imposed in their workers (congestion) as well as the costs imposed to the rest of the society (i.e., congestion, pollution). In this paper, a study of a strategy to influence the demand generator by relaxing the arrival constraints is presented. Bi-level programming models are used to investigate the equilibrium reached from the firm-workers interplay which helps to explain how the market failure arises. The evaluation includes the use of incentives to induce the shift to less congested periods and the case of the social system optimum in which a planner objective is incorporated as a third agent usually seeking to improve social welfare (improve productivity of the firm while at the same time reduce the total system travel time). The later is used to show that it is possible to provide a more efficient solution which better off society. A numerical example is used to (1) show the nature of the market failure, (2) evaluate the social system optimum, and (3) show how a congestion tax or an optimal incentive can help to correct the market failure. The results also corroborate that

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these mechanisms are more likely to be more efficient when firms face little production effects on time and workers do not high opportunity costs for starting at off peak periods.

Keywords Peak congestion · Rescheduling of work · Market failure · Bi-level models · Stackelberg · Dynamic traffic assignment

1 Introduction

There is the widespread view that the peak hour/period traffic is nothing but the result of traveler decisions (e.g., Ott et al. 1980; Moore et al.1984). Under this perspective, travelers decide what their preferred departure times and times of travel are in response to expectations of travel conditions, and from that, some sort of equilibrium (static or dynamic) emerges. Implicit in this assumption is the idea that the traveler has the possibility to reschedule, modify, and even cancel the trip. At the heart of this issue are the differences between compulsory and non-compulsory trips. While in the latter, the traveler has great liberty to change travel patterns; the same cannot be said about the former because of the existence of more rigid travel constraints. This is because in the case of compulsory trips, there are powerful external agents (e.g. employers, educational institutes) that mandate the arrival and departure times almost without feedback from their customers (e.g. workers, students). As a result, assuming that travelers could react by changing time of travel decisions, it requires taking proper consideration of the corresponding travel constraints. In the case of non compulsory trips, the constraints are not likely to be very tight; while in the compulsory case, they are likely to severely restrict what this traveler could do in reaction to either traffic conditions or public sector interventions like tolls. This in turn poses a major challenge because in the peak hours and peak periods -where the use of pricing is likely to be justified- the bulk of this traffic (passenger and freight) is of a compulsory nature, i.e., work trips. The behavior of the travelers is therefore controlled by the demand generator while the travelers have little or no means to influence what the demand generators do.

In work trips and work related trips, it is the firm who sets up the starting time (which usually is during peak hours) without with little or not consideration of the implications and impositions to the workers (the users of the transportation system). Workers end tied to the firm's decision suffering from the additional costs of traveling during peak periods. The Evaluation Study of the Port Authority of New York and New Jersey's Time of Day Pricing Initiative (Holguín-Veras 2005, 2011) seems to support this idea. The study found that the decisions for traveling at certain time are usually constrained by their work schedules (about 87 percent of AM peak users and 67 percent of the PM peak users) with a relatively narrow windows of flexibility (see Table 1) of about 20 minutes for early departure or arrival, and about 14 minutes for late departure or arrival. Moreover, according to the report, about 46.6 percent of the work trip respondents had no flexibility at all in arriving late. Flexibility was higher if workers decided to arrive early, while 19 percent of workers had no flexibility at all in this case. Only roughly 10 percent of trips reported a flexibility of 30 minutes

Table 1 Mean (standard deviation) of time of travel flexibility in work related trips

Time of travel flexibility	Car	Transit
Earlier departure*	19.0 (24.5)	17.9 (9.5)
Later departure*	14.7 (20.8)	14.9 (17.9)
Earlier arrival**	20.4 (24.0)	18.3 (10.3)
Later arrival**	12.3 (19.0)	9.1 (7.5)

* How earlier or later they were willing to depart from the origin and still meet their travel constraints) and arrival flexibility

** How earlier or later they were willing to arrive at their destinations and still meet their travel constraints

or more. These results are similar to those found by Emmerink and van Beek (1997) who used data collected in 1992 in The Netherlands. They have found even larger percentages: 67 percent of the respondents indicated that their employers do not allow for any flexibility at all.

This implies that most travelers cannot simply wait for the non-peak period due to arrival time constraints, which are often tight and inflexible. This implication is important because some strategies, such as congestion pricing, can be only applicable to a small percentage of users during peak hours who have certain flexibility in their schedules. On the contrary, they are not generally applicable to most workers as their schedules are not flexible. Flexibility in arrival time is, as pointed out by Emmerink and van Beek (1997), a "...necessary condition for implementing congestion pricing..." and therefore a necessary condition for implementing demand management policies that seek to alter travel behavior patterns.

This relatively little or no flexibility found indicates that users have constraints imposed by jobs that make it difficult to shift their time of travel. Firms tend to benefit from having their workers on duty during the same hours as well as at the same time as other companies (competitors or clients), imposing the working schedules to a large portion of their workers. As Jones et al. (1977) noticed, there is an asymmetric firm-worker's relationship with a firm acting as the dominant agent: "Most choices to travel in the peak period should be viewed as a joint decision of the individual and the employer because the "choice" of travel time is determine by work schedules...Employers are likely to have far more discretion that their employees in scheduling work hours...Most employees face a Hobson's choice[1] between unemployment and work schedule conformance..." And, when such type of asymmetry is present, it can be expected that a market failure arises.

A market failure is a situation in which the allocation of goods or services is not efficient. It is usually associated with the presence of public goods, information asymmetry, non-competitive market, or externalities (Ledyard 2008). In such situations the first two theorems for welfare do not hold (Mas-Colell et al. 1995) and the market cannot reach an efficient solution. The evidence presented earlier has shown that the current scheme favors the firm who sets up the starting time (which usually is

[1] A "take it, or leave it" option of the single offer that is given (Addition and Steel 2012).

during peak hours) with little or no consideration of the implications and impositions to the workers, who are the users of the transportation system. If workers are tied to a firm's decision that sets up standard schedules (fixed or tight), they will suffer from the additional costs of traveling during peak periods. Thus the failure mechanism in this scheme is that the presence of the private utility of the dominant agent (the firm) prevents the society to reach a social optimum that improves social welfare. This private utility does not account for the additional costs imposed in their workers as well as the costs imposed to the rest of the society in terms of traffic congestion and pollution, making this scheme inefficient for society.

The presence of this market failure also justifies public sector intervention. Economics offers two types of instruments for governments to address the problem for transport market failures (congestion, road and environmental damage, road accidents, etc): command-and-control and incentive based policies (Santos et al. 2010). The former are regulations imposed by the government to consumers and users to change their behavior. Examples of these regulations are emission standards, parking restrictions, among others. Incentive based models are economic mechanisms in which targeted agents receive incentives to alter directly their private utility or cost that result from their behavior. Examples of these mechanisms are taxes such as congestion pricing, subsidies, and incentives (Santos et al. (2010) provide a comprehensive description of the incentives used in transportation and their policy implications). In the particular case of the peak congestion, the key is to modify the behavior of the dominant agent (the firm) that creates the demand making it more attractive to reschedule workers to off-peak periods. The intervention is therefore more suitable for an incentive focused on the dominant agent rather than imposing charges to the dominated ones. Such type of approach has been applied in the implementation of a shift in freight traffic from regular hours to off-peak hours through incentives (Holguín-Veras et al. 2005). In these cases, financial incentives compensate the inconvenience caused by this practice in receivers as shown in Holguín-Veras (2008, 2011) and Holguín-Veras et al. (2006). The success of such type of approach is well documented in Holguín-Veras et al. (2011).

The purpose of this paper is to study the nature of the market failure related to the problem of tight and restrictive arrival times in workers imposed by firms. This is done by modeling the firm-worker interplay providing an alternative work schedule. The aim is to flatten the peak hours by assigning some of the demand to the less congested periods, and therefore reducing the peak demand. Past initiatives and pilot experiments have claimed that fewer work trips are expected during the peak hours (Maric 1977; Guiliano 1990; O'Malley and Selinger 1973; Owens and Warmer 1973). This not only reduces costs in society (e.g., congestion, pollution) but also helps to minimize the physical requirements. However, formal experiments with this policy continued in the 1980s but its use has declined over time. In order to make this strategy sustainable it is necessary to go in depth in understanding the behavior of the agents (firms and workers) involved when work activities are rescheduled as well as in translating such behavior into a mathematical framework with the correct interaction between agents. In the model proposed in this paper, a mathematical framework captures such interaction at a network level. This includes (1) departure time choice, (2) route choice, (3) network interactions (i.e., flow propagation), and (4) a general production effects on the firm model based on arrivals. The main assumption is that the firm, affected by production effects, seeks to maximize their utility. This is addressed using an indirect measure of the productivity based on the "Theory of Marginal Productivity" that has been used in the only two studies that have sought to measure production effects as a function of work starting times (i.e., Wilson 1988; Gutiérrez-i-Puigarnau and Ommeren 2012). Once the firm decides the work schedules, the workers react by adjusting their departure time and route choice behavior balancing them with their actual arrival time until a Dynamic User Equilibrium (DUE) is achieved. The actual arrival time thus affects the firm productivity and the negotiation is repeated with firm the firm acting as the leader and the workers as followers in a Stackelbergh equilibrium. This is set up in a bi-level programming framework that not only helps to show the market failure but also it is useful to evaluate corrective measures based on incentives or based on congestion taxes.

The paper is organized as follows: Section 2 presents the analysis of the agents involved (firms and workers) and the mathematical models that captures their behavior. Section 3 presents the bi-level models developed used in the evaluation. Section 4 presents the solution algorithm. Section 5 presents an illustrative example that uses real data to capture the production effects on firms. Finally Section 6 presents the conclusions.

2 Modeling the Behavior of Firms and Workers

The modeling of firm-workers interplay in a dynamic network proposes a challenge from the network modeling perspective because a proper consideration of the asymmetric relationship between the agents needs to be considered. This departs from the symmetric distribution of power assumed in Arnott et al. (1990b, 2005), Henderson (1981), Mun and Yonekawa (2006) and Yoshimura and Okumura (2001). In addition, the nature of the problem requires a proper consideration of the dynamics of the traffic model. Dynamic User Equilibrium (DUE) and Dynamic Traffic Assignment (DTA) are required to quantify the effects and behavior of the users of the transportation system (workers). This addition overcomes the limitations of the single bottleneck models (Vickrey 1969; Arnott et al. 1990a), used in Arnott et al. (1990b, 2005), Henderson (1981), Mun and Yonekawa (2006) and Yoshimura and Okumura (2001) to model two rescheduling of work policies: staggered work hours or flextime, providing a model that balances the firm's objective, traffic dynamics, and traveler behavior reality in a single framework.

2.1 Modeling Firm's Behavior by Approximating Production Effects through Wages

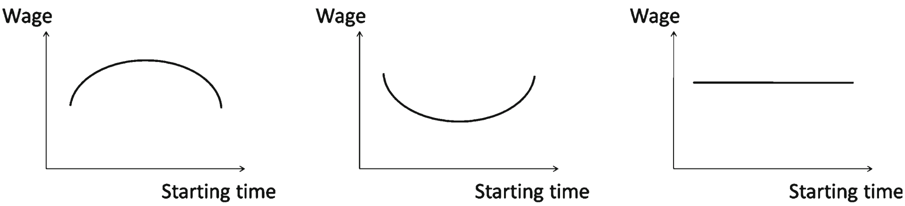
There is a general consensus regarding that when firm reschedule workers (i.e., in staggered work hours) a firm faces productions effects. However, there is no consensus whether these effects are positive or negative. In one hand, Lucas (1970) argues that workers starting at an off-peak time may be more productive because they face less competition for the use of the firm's capital and also suffer less fatigue by traveling at lower congestion such as in Shepard et al. (1996). On the other hand, Henderson (1981) and Arnott et al. (2005) claimed that agglomeration economies (Mills 1967; Fujita and Thisse 2002) impose negative production effects because having all workers at the same times in the same days allows interaction within and across firms. Such interactions are lost if workers are, for instance staggered (e.g., O'Malley and Selinger 1973; Sloane 1975; Guiliano and Golob 1990).

This lack of consensus is recognized by Arnott et al. (2005) who mention that it is because there are no empirical studies on the pattern of work starting times to infer the intra-organizational productivity. However, production effects can also be addressed in an indirect way through the "Marginal Productivity Theory of Wage" originally stated by Von-Thünen and later developed by Walras (1954). This theory states that, in a competitive market, workers maximize their utility, firm maximize their profit, and wages equal marginal productivity. More specifically, wage is equal to the marginal revenue productivity of labor (Gowdy 2009). The marginal revenue productivity of labor (MRP_L) is the addition made to the total revenue *TR* by employing one more unit of a worker *L* or

$$MRP_L = \frac{\Delta TR}{\Delta L} = w \tag{1}$$

If workers arriving at off-peak intervals (i.e., workers are staggered, SW) receive higher compensation than workers arriving at the peak (P) or $w_{SW} > w_P \Rightarrow \frac{\Delta TR_{SW}}{\Delta L_{SW}} > \frac{\Delta TR_P}{\Delta L_P}$, this would be indicative that they are being more productive as Lucas (1970) suggests. If, on the other hand, the opposite is true, then workers prefer to start at the peak-hour because that makes them more productive, and therefore receive more compensation or $w_{SW} < w_P \Rightarrow \frac{\Delta TR_{SW}}{\Delta L_{SW}} < \frac{\Delta TR_P}{\Delta L_P}$.

Therefore, one proxy for the productivity of the firm (and its corresponding value in utility) can be obtained by the estimation of the wages paid by the firm. This is how two empirical studies tried to address the issue of measuring production effects in terms of work starting times in the past. Wilson used 581 survey data observations from Singapore collected by the World Bank in 1975 to estimate an econometric model of monthly income as a function of starting time (Wilson 1988). His results seem to match a strongly U-inverse shape function as in Fig. 1a. This means that workers starting at peak times are paid much more than those starting at the non peak hours. But the degree of the concavity of the function depends on the occupational group, e.g., the sales group is significantly less concave than the professional group. More recently, Gutiérrez-i-Puigarnau and Ommeren (2012) have conducted an analysis using 12,231 observations for 9,210 workers for years 2004 to 2006 in Germany.



(a) Inverse U-shape (b) U-shape (a) Flat-shape

Fig. 1 Possible production effect functions by starting time

The approach used is an econometric model to estimate the effect of start time on wages using the logarithmic wage as a function of start time, individual attributes, and job characteristics. They found that the general relationship between starting times and wages is slightly U-inverse which implies a weak (or no) relationship between wages and start time. Their results imply that the start time of workers has relatively little effect on wages and therefore productivity and workers' disutility, suggesting also certain benefit of staggered work hours in society.

One explanation for such difference in the magnitude of the effects between Wilson (1988)'s and Gutiérrez-i-Puigarnau and Ommeren (2012)'s studies can be found in the year when each study was performed. The last decade has shown greater advances in technology (e.g., telecommuting, cellphones, smartphones) which allow for better communication with clients and co-workers without having to be present in the office. Such technologies did not even exist at the time when Wilson (1988) performed his study. Therefore it can be expected certain reduction in the production effects on time in recent years. For the models presented in this work, the following general quadratic expression (2) claimed by both studies will be the basis for modeling the firm's objective function:

$$\ln(w) = \beta_0 + \beta_1 k + \beta_2 k^2 \tag{2}$$

where w represents the wage, and the parameters β_0, β_1, and β_2 help to capture the production effects with respect to the arrival time k.

The productivity (utility) function can be thus approximated using equation (2). However such values have to be converted into actual wage values. This issue is solved by simply using the exponential value of the equation obtained. Therefore, the resulting production effects function (3) is:

$$\exp\left(\beta_0 + \beta_1 k + \beta_2 k^2\right) \tag{3}$$

2.2 Dynamic User Equilibrium with Departure Time Choice for Multiple Arrival Times: A Nonlinear Complementarity Problem (NCP) model

For the purpose of modeling work rescheduling, vehicles represent travelers with different desired arrival times. Therefore, a DUE is required in order to define the rules used by travelers when deciding their routes and departure times given that they need to arrive at a specific time or within a specific time threshold. In this case, the definition proposed can resemble a multi-class DUE (Bliemer and Bovy 2003), in the sense that each class can be defined as the group of drivers that have the same arrival time. Therefore, for the sake of this paper, a class will refer to all drivers with the same desired arrival time instead of a vehicle class. Such two choices (departure and route choice) can be presented as Nonlinear Complementarity Problem (NCP) (Facchinei and Pang 2003a, b) extending the link-node approach developed by Ban et al. (2008) to consider multiple groups of user with different starting times. In this case, assuming that a transportation network can be represented as a directed graph $G(N, A)$ where N is the set of nodes and A the set of links, the total inflow rate of class m to link a towards a destination s at the beginning of the interval k can be now denoted as u_{as}^{km} with $a \in A$, $s \in N$, $k \in K$, and $m \in M$. The travel time on link a at interval k, $\tau_a^k(u)$ is a function of the total inflow on link a rate $u = \left(\sum_m u_{as}^{km}\right)_{\forall a,s,k}$. The DUE condition in this case becomes into:

> "If, from each decision node to every destination node at each instant of time, the actual travel times for all the routes that are being used by each "class" (or group of drivers with the same work schedule) are equal and minimal, then the dynamic traffic flow over the network is in a travel time based dynamic user equilibrium (DUE) state."

Under this condition, a decision node with respect to a destination can be any node that generates trips (i.e., an origin) or is traversed by flows heading to the destination and the condition for DUE can be developed as flow conservation of inflow/exit flow, the route choice, the travel time computation (and the equilibrium travel times) and departure time choice. Notice that, vehicles are not restricted to depart at the same time to satisfy the departure time condition. Then each class represents not a class of vehicles but a class or group of workers that have the same work schedule (work starting time) $\bar{k}_m$.

The next two sections summarizes the formulation in Yushimito et al. (2013) that embeds the previous condition into a dynamic user equilibrium model. This formulation uses the notation included in Appendix A.

2.2.1 Departure Time Choice

The departure time choice can be established based on a utility function that depends on travel time costs and schedule delay. Different from models like those in Arnott et al. (2005) or Henderson (1981), workers decisions are handled by the total disutility of travel which can be monetized through the value of time. The disutility of travel is determined by the departure time choice. Thus, in this case start times are the result of the effects of the start times on workers' utility due to inconvenient schedule costs (schedule delay) and longer commuting times, as well as the effect of start time on productivity. Thus, the behavior is modeled using the following scheme: workers decide when to depart based on their considerations of their travel time, penalty function for early/late arrival and the inconvenience of a new starting time. In other words, if the class m worker starts from the origin at time k, it has a disutility function of the form.

$$\Psi_{is}^{km} = \pi_{is}^{km} + \phi + \Lambda \tag{4}$$

where the first term refers to the travel time. The third term is the inconvenience cost due to the rearrangement of non-work related activities, which has the form $\Lambda = \xi \left(k - \bar{k}_m\right)^2$, where ξ penalizes the trips that are deviating from the preferred arrival time $\bar{k}_m$ in a quadratic fashion.

The second term ϕ represents the early/late arrival delay with respect to the starting time $\bar{k}_m$, a symmetric function can be of the form $\phi = \alpha \left(k\Delta + \pi_{is}^{km} - \bar{k}_m\right)^2$. However, it is often argued that the late arrival penalty should be much larger than the early arrival penalty (Small 1982). In this case, a different coefficient can be used as shown in Ran and Boyce (1996) and Ran et al. (1996) by having two different parameters α_1 (early) and α_2 (late). Moreover, it can be argued that there is no penalty if the worker arrives during a certain time threshold σ. This is not only a reasonable assumption, but also accounts for some flexibility, which both in practice (Transportation Research Board 1980) and for a single bottleneck (Arnott et al. 1990b) problem has been shown to produce better effects. The resulting scheme is depicted in Fig. 2 and can be expressed in mathematical terms as $\phi = \max(\nu, 0)$, where $\nu = \max\left(\alpha_1\left(\bar{k}_m - \sigma - \pi_{is}^{km}\right), \alpha_2\left(\pi_{is}^{km} - \bar{k}_m - \sigma\right)\right)$.

Defining an additional variable η, we can define the following functions, in terms of the vectors $u = \left(u_{is}^{km}\right)_{\forall a,s,k,m}$, $\pi = \left(\pi_{is}^{km}\right)_{\forall i,s,i\neq s,k,m}$, $d = \left(d_{is}^{km}\right)_{\forall i,s,k,i\neq s,m}$, $\mu = \left(\mu_{is}^{m}\right)_{\forall i,s,k,i\neq s}$ and variables ϕ, η:

$$\Phi_\phi(u,\pi,d,\mu,\phi,\eta) = \phi - \eta - \alpha_1\left(\bar{k}_m - \sigma - \pi_{is}^{m}\right) \tag{5}$$

$$\Phi_\eta(u,\pi,d,\mu,\phi,\eta) = \eta + \alpha_1\left(\bar{k}_m - \sigma - \pi_{is}^{m}\right) - \alpha_2\left(\pi_{is}^{m} - \bar{k}_m - \sigma\right) \tag{6}$$

and the schedule delay can be expressed as a the following complementarity constraints:

$$0 \le \phi \perp \Phi_\phi(u,\pi,d,\mu,\phi,\eta) \ge 0 \tag{7}$$

$$0 \le \eta \perp \Phi_\eta(u,\pi,d,\mu,\phi,\eta) \ge 0 \tag{8}$$

Further denote the minimum disutility between an origin node i and a destination node s for the group (class) m (for all time intervals) as μ_{is}^{m}, the departure choice condition for workers in that class can be expressed by the following two conditions:

$$d_{is}^{km} \ge 0, \text{ if } \Psi_{is}^{km} = \left(\pi_{is}^{km} + \phi + \Lambda\right) = \mu_{is}^{m} \tag{9}$$

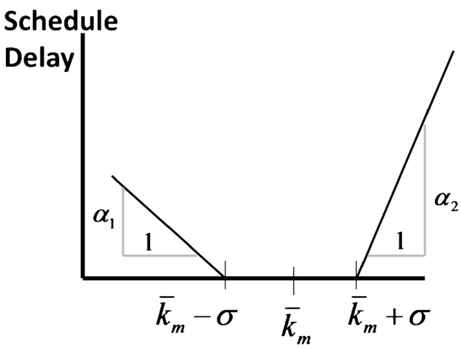
$$d_{is}^{km} = 0, \text{ if } \Psi_{is}^{km} = \left(\pi_{is}^{km} + \phi + \Lambda\right) > \mu_{is}^{m} \tag{10}$$

In other words, travelers are trying to minimize their disutility when choosing departure times. This can be set up as the following complementarity constraint

$$0 \le d_{is}^{km} \perp \Phi_d(u,\pi,d,\mu,\phi,\eta) \tag{11}$$

where $\Phi_d(u,\pi,d,\mu,\phi,\eta) = \left(\pi_{is}^{km} + \phi + \Lambda\right) - \mu_{is}^{m}$, $\forall\, k, m, i$.

Fig. 2 Asymmetric schedule delay function



In addition, the total departures among all intervals have to sum up to the total departures per group; and the total departures per group have to sum up to the total number of workers to guarantee the flow conservation between the departures and the total demand:

$$\sum_k d_{is}^{km} = dd_{is}^m \tag{12}$$

Making $\Phi_\mu(u, \pi, d, \mu, \phi, \eta) = \sum_k d_{is}^{km} - dd_{is}^m$, this equations can be used to write a complementarity that represents the condition that the flow conservation of departures is satisfied whenever the disutility is greater than zero:

$$0 \leq \mu_{is}^m \perp \Phi_\mu(u, \pi, d, \mu, \phi, \eta) \geq 0 \tag{13}$$

provided that $\sum_m dd_{is}^m = Q_{is}$

2.2.2 Route Choice and Dynamic User Equilibrium

Previous research did not explicitly consider route choices due to the simplistic network structure (i.e., one OD pair, one route). To model the congestion effect, Henderson (1981) assumes that congestion is a function of the number of commuters departing at each time, which also implies that flow departing at different times do not interact. To overcome this problem, Chu (2003) reformulated the problem by assuming that the travel time for a commuter is determined by the arrival flow rather than by the departure flow. In other words, the travel time is defined by the number of travelers arriving together rather than the number of travelers departing together. However, his model uses a single link and single OD pair that is comparable with Vickreys single bottleneck model (which was shown to be the limit of the reformulated Henderson approach when the elasticity to travel delay goes to infinity). Given this simplistic network structure (single OD pair and single), network effects are not considered for any of these models. As the departure choice has been already discussed, the route choice model is described.

The proposed model adopts a predictive DUE concept which can be found in Heydecker and Verlander (1999) and Ban et al. (2008). In addition to the notation shown in Appendix A, we define $e_a^k(\mathbf{u}) = (k-1)\Delta + \tau_a^{k-1}(\mathbf{u})$ defined as the exit time of vehicles entering link a at the beginning of time k, which is a function of the link inflow vector $\mathbf{u} = \left(\sum_m u_{as}^{km}\right)_{\forall k,a,s}$. Further, denote l_a, h_a, the tail (starting) and head (ending) nodes of link a. The conservation of flow can be represented using the discretization scheme introduced in Ban et al. (2008). For any time interval k this can be represented by the relationship between the exit flow and inflow rates

$$\sum_{a \in A(i)} u_{as}^{km} = d_{is}^{km} + \sum_{a \in B(i)} v_{as}^{km}, \ \forall\, i, s, i \neq s, k \tag{14}$$

This representation requires a way to represent the flow propagation. Following Ban et al. (2008), this requires the definition of two indicator functions

$\lambda_a^{1,k'}(\mathbf{u})$, $\lambda_a^{2,k',k}(\mathbf{u})$ that are function of the time interval length Δ and the travel time function τ which depends on the total inflow.

$$\lambda_a^{1,k'}(\mathbf{u}) = \frac{\Delta}{\tau_a^{k'}(\mathbf{u}) - \tau_a^{k'-1}(\mathbf{u}) + \Delta} \tag{15}$$

$$\lambda_a^{2,k',k}(\mathbf{u}) = \frac{\tau_a^{k'}(\mathbf{u}) + \left(k' + 1 - k\right)\Delta}{\tau_a^{k'}(\mathbf{u}) - \tau_a^{k'-1}(\mathbf{u}) + \Delta} \tag{16}$$

With these two parameters defined, it is possible to define the exit flow as a function of the inflow:

$$v_{as}^{km} = \sum_{\substack{k' e_a^{k'}(\mathbf{u}) \leq \\ (k-1)\Delta < e_a^{k'+1}(\mathbf{u})}} \left(\lambda_a^{2,k',k}(\mathbf{u}) v_{as}^{k'm}\left(e_a^{k'}\right) + \left(1 - \lambda_a^{2,k',k}(\mathbf{u})\right) v_{as}^{k'm}\left(e_a^{k'}\right)\right) \tag{17}$$

where, $v_{as}^{km}\left(e_a^{k'}\right) = u_{as}^{k'm}\lambda_a^{1,k'}(\mathbf{u})$ and equation (14) can be finally stated as

$$\begin{aligned}\Phi_\pi\left(u, \pi, d, \mu, \phi, \eta\right) = &\sum_{a\in A(i)} u_{as}^{km} - d_{is}^{km} \\ &- \sum_{a\in B(i)} \sum_{\substack{k' e_a^{k'} (k-1)\Delta \\ < e_a^{k'+1}(\mathbf{u})}} \left(\lambda_a^{1,k'}(\mathbf{u})\lambda_a^{2,k',k}(\mathbf{u}) u_{as}^{k'm}\right. \\ &\left. + \lambda_a^{1,k'+1}(\mathbf{u})\left(1 - \lambda_a^{2,k',k}(\mathbf{u})\right) u_{as}^{k'+1}\right) \geq 0, \\ &\forall\, i, s, i \neq s, k'\end{aligned} \tag{18}$$

or simply

$$\Phi_\pi\left(u, \pi, d, \mu, \phi, \eta\right) = \sum_{a\in A(i)} u_{as}^{km} - d_{is}^{km} - \sum_{a\in B(i)} v_{as}^{km} \tag{19}$$

Consequently, flow conservation can then be expressed as the following complementarity condition:

$$0 \leq \pi_{is}^{km} \perp \Phi_\Pi\left(u, \pi, d, \mu, \phi, \eta\right) \geq 0,\ \forall i, s, i \neq s, k' \tag{20}$$

The DUE route choice condition is a dynamic extension to the Wardrop's first principle (Wardrop 1952) for the static case. Defining an indicator function $\lambda_a^{3,k,l}(\mathbf{u})$, Ban et al. (2008) have shown that this condition can be represented using the a complementarity equation which was later extended to account for multiple classes by Yushimito et al. (2013) as follows:

$$0 \leq u_{as}^{km} \perp \Phi_u\left(u, \pi, d, \mu, \phi, \eta\right) \geq 0,\ \forall a, s, k', m \tag{21}$$

where

$$\Phi_u\left(u, \pi\right) = \tau_a^k(\mathbf{u}) + \sum_{l-1 \leq e_a^{k+1}(\mathbf{u})/\Delta < i} \lambda_a^{3,k,l}(\mathbf{u})\pi_{h_a s}^{l-1} + \left[1 - \lambda_a^{3,k,l}(\mathbf{u})\right]\pi_{h_a s}^{l} - \pi_{l_a s}^{k}, \forall a, s, k' \tag{22}$$

and

$$\lambda_a^{3,k,l}(\mathbf{u}) = \frac{l\Delta - e_a^{k+1}(\mathbf{u})}{\Delta} = \left(l - k - \frac{\tau_a^k(\mathbf{u})}{\Delta}\right), \forall a, k, l; (l-1) \leq \frac{e_a^{k+1}(\mathbf{u})}{\Delta} < l \tag{23}$$

2.2.3 *Dynamic User Equilibrium Formulation*

The resulting model is the following Nonlinear Complementarity Problem (NCP) which incorporates both the departure time choice and the route choice see also Yushimito et al. (2013). The model seeks to find the vectors u_{as}^{km}, π_{is}^{km}, d_{is}^{km}, μ_{is}^{m}, and variables ϕ, η such that

$$0 \leq u_{as}^{km} \perp \Phi_u(u, \pi, d, \mu, \phi, \eta) \geq 0, \forall a, s, k, m \tag{24}$$
$$0 \leq \pi_{is}^{km} \perp \Phi_\pi(u, \pi, d, \mu, \phi, \eta) \geq 0, \forall i, s, i \neq s, k, m \tag{25}$$
$$0 \leq d_{is}^{km} \perp \Phi_d(u, \pi, d, \mu, \phi, \eta) \geq 0, \forall k, m, i, s \tag{26}$$
$$0 \leq \mu_{is}^{m} \perp \Phi_\mu(u, \pi, d, \mu, \phi, \eta) \geq 0, \forall i, s, m \tag{27}$$
$$0 \leq \phi \perp \Phi_\phi(u, \pi, d, \mu, \phi, \eta) \geq 0 \tag{28}$$
$$0 \leq \eta \perp \Phi_\eta(u, \pi, d, \mu, \phi, \eta) \geq 0 \tag{29}$$

with $\sum_m dd_{is}^m = Q_{is}$.

3 Bi-Level Formulation of Alternative Work Arrival Times

This section presents mathematical models that captures the interaction of a firm and its workers at a network level with a more rigorous treatment of the time dimension. This includes the behavior of the agents already described in Section 2 as well as the asymmetric interaction between them that was explained in the introduction. Thus in the model, first the firm, by being affected by production effects, seeks to maximize their utility. Second the workers react by adjusting their departure time and route choice behavior until a Dynamic User Equilibrium (DUE) is achieved. As the firm has more discretion in selecting the starting time of the workers, the firm proposes some starting times and the workers will react based on how they can adjust their departure or find a better route choice to arrive at that starting time. This implies that the workers will respond and some will accept and some will reject the starting time. This interaction scheme assumes that at the end firm and workers reach a certain equilibrium condition in their negotiation. However, the firm maintains its role as the leader. In other words, the firm will make the first move and the workers will follow their decisions as in a Stackelberg game (Von Stackerlberg 1952) which, mathematically, can be formulated as a bi-level model (Luo et al. 1996). Therefore, the model uses a bi-level formulation to investigate the effects of relaxing arrival constraints induced by incentives. The evaluation includes the case of the social system optimum which will be the ultimate goal of a planner seeking to improve social welfare.

3.1 Bi-Level General Model

The assumption that the firm can receive a feedback from workers sets up a framework for a bi-level model in which the firm act as the leader and workers as followers of the firm's decision about the starting time. The firm behavior is determined by their ability to respond to the production effects rescheduling work arrival times. In this section, the "Marginal Productivity Theory of Wage" is used to model the production effectson firms while the DUE condition and DTA model described in Section 2 is the basis for modeling the behavior of the worker. Thus the model is uses the general production effect based on the quadratic expression (3) in the upper level, and the NCP (24–29) used to capture the multiple arrival times DUE for the traffic and workers behavior as the lower level problem. The formulation is shown below:

$$\text{maximize} \quad \sum_{k=1}^{T} \left[\gamma . \exp\left(\beta_0 + \beta_1 k + \beta_2 k^2\right) \sum_{i}^{I} v_{is}^{k} \right] \tag{30}$$

subject to Eqs. 24–29 and $\sum_m dd_{is}^m = Q_{is}, \forall i, s$

In the formulation, the objective function (30) seeks to capture the benefit of the firm as function of the arrivals v_{is}^k at interval k using the function $\beta_0 + \beta_1 k + \beta_2 k^2$ which is nothing but the benefit that a firm can be obtained from a worker arriving at time k. In other words the benefit of the firm is based on the actual arrival of its workers and the benefit provided by each worker arriving at a particular time. This function can be approximated through the use of an estimation of the logarithm of the wages by starting time as in Gutiérrez-i-Puigarnau and Ommeren (2012). Such values have been converted into actual wage values by using the exponential and then into travel time using a parameter γ to convert travel time into time units (which is nothing but the inverse of the value of time) in order to have the same units as the NCP problem. The constraints represented by Eqs. 24 to 29 are based on the traffic model described earlier.

3.2 Bi-Level Formulation with Incentives for Non Peak Arrivals

The model described above can be adapted to handle incentives to compensate the firm assigning workers in the off-peak periods (i.e., staggering their arrivals). This incentive has to be related to the effects in the congestion by spreading the arrivals away from the peak-period. Therefore, it can be assumed that the firm receives an incentive if its workers start work at alternative schedules; otherwise, no incentive is provided. One way to model this is to choose an incentive as a function of the movement with respect to the peak period (which is assumed the preferred arrival period for the firm):

$$\text{Incentive} = \theta_1 \sum_{k<\bar{k}_0-\sigma} \sum_{i}^{I} v_{is}^{k} + \theta_2 \sum_{k>\bar{k}_0+\sigma} \sum_{i}^{I} v_{is}^{k} \tag{31}$$

Here θ_1 and θ_2 can be understood as the amount of the incentive per worker arriving before the peak-period time $\bar{k}_0 - \sigma$ and after the period time $\bar{k}_0 + \sigma$, with σ being the tolerance of arrival. This amount is applied to all workers arriving at schedule before the peak period time $\sum_{k<\bar{k}_0-\sigma} \sum_{i}^{I} v_{is}^{k}$, and to all workers arriving after the peak period time $\sum_{k>\bar{k}_0+\sigma} \sum_{i}^{I} v_{is}^{k}$ defined through the tolerance σ in arrival with respect to the specific starting time $\bar{k}_0$.

The equivalent MPEC formulation is therefore:

$$\text{maximize} \sum_{k=1}^{T} \left[\gamma . \exp\left(\beta_0 + \beta_1 k + \beta_2 k^2\right) \sum_{i}^{I} v_{is}^{k} \right] + \theta_1 \sum_{k<\bar{k}_0-\sigma} \sum_{i}^{I} v_{is}^{k} + \theta_2 \sum_{k>\bar{k}_0+\sigma} \sum_{i}^{I} v_{is}^{k} \tag{32}$$

subject to Eqs. 24–29 and $\sum_m dd_{is}^m = Q_{is}, \forall i, s$

3.3 Bi-Level Formulation of the Social System Optimum

In transportation, the system optimum is defined as the solution where the assignment assumes that travelers will cooperate to make their choices or a planner will decide for them to benefit the whole system instead of their own individual benefits. However, in such solution the planner only considers the transportation system. A more comprehensive objective is one that includes not only the transportation network but also the firm's objective, as both the transportation and firm interact in work related trips. The objective is therefore to reach the best distribution of starting times that gives an optimal utility for the firm as well as a reduction in the total congestion. As it has also been shown in Yushimito (2011), these two objectives may conflict to each other. Firms have production effects because firms prefer to have a more concentrated distribution of its workers, while workers usually are trying to avoid traffic unless they have additional reasons different from travel time costs. However, there is a third agent that is usually not considered in the analysis, the planner who wants to reduce the overall congestion of the transportation system. Therefore any solution should also consider the planner whose objective should be the social system optimum which the situation in which the objectives of all these agents are maximized: firms and workers at the same time.

Under these considerations, the social system optimum (SSO) needs to include a model where the objective of the planner is subject to the the objective of the firm which provides the input for the traffic and worker's behavioral model. That is, a hierarchical model. However, the advantage of defining the firm's objective in terms of the arrival simplifies the number of levels in the model because the social system optimum can be directly addressed in the objective function:

$$\text{maximize} \sum_{k=1}^{T} \left[\gamma \exp\left(\beta_0 + \beta_1 k + \beta_2 k^2\right) \sum_{i}^{I} v_{is}^{k} \right] - \sum_{a,k} \tau_a^k(u) u \tag{33}$$

where the first term is again the utility or productivity function obtained by approximating the wages in terms of the arrivals and the second term is the total system travel time in terms of the inflow. The resulting model is:

$$\text{maximize} \sum_{k=1}^{T} \left[\gamma \exp\left(\beta_0 + \beta_1 k + \beta_2 k^2\right) \sum_{i}^{I} v_{is}^{k} \right] - \sum_{a,k} \tau_a^k(u)u \tag{34}$$

subject to Eqs. 24–29 and $\sum_m dd_{is}^m = Q_{is}, \forall i, s$

4 Solution Algorithm

4.1 Relaxation Scheme

The solution approach adopted in tis paper is the relaxation scheme proposed by Ban and Liu (2009) for the bi-level dynamic congestion pricing model where the bi-level formulation is converted into a single level nonlinear programming (NLP) problem.This approach is based on reformulating the complementarity constraints as strict complementarity constraints. Thus, in addition to the non negativity constraints of the variables and equations in the left and right hand side in Eqs. 24–29, the following equations need to be added to have a single level problem:

$$u_{as}^{km}\Phi_u\left(u, \pi, d, \mu, \phi, \eta\right) \leq \epsilon, \forall a, s, k, m \tag{35}$$

$$\pi_{is}^{km}\Phi_{pi}\left(u, \pi, d, \mu, \phi, \eta\right) \leq \epsilon, \forall k, m, i, s \tag{36}$$

$$d_{is}^{km}\Phi_d\left(u, \pi, d, \mu, \phi, \eta\right) \leq \epsilon, \forall k, m, i, s \tag{37}$$

$$\mu_{is}^{m}\Phi_{mu}\left(u, \pi, d, \mu, \phi, \eta\right) \leq \epsilon, \forall i, s, m \tag{38}$$

$$\phi\Phi_{\phi}\left(u, \pi, d, \mu, \phi, \eta\right) \leq \epsilon, \forall i, s, m \tag{39}$$

$$\eta\Phi_{\eta}\left(u, \pi, d, \mu, \phi, \eta\right) \leq \epsilon, \forall i, s, m \tag{40}$$

The resulting relaxed problems are the following NLP problems:

Relaxed general model:

$$\text{maximize} \sum_{k=1}^{T} \left[\gamma . \exp\left(\beta_0 + \beta_1 k + \beta_2 k^2\right) \sum_{i}^{I} v_{is}^{k} \right] \tag{41}$$

subject to

$$u_{as}^{km} \geq 0, \forall a, s, k, m \tag{42}$$

$$\Phi_u\left(u, \pi, d, \mu, \phi, \eta\right) \geq 0, \forall a, s, k \tag{43}$$

$$u_{as}^{km}\Phi_u\left(u, \pi, d, \mu, \phi, \eta\right) \leq \epsilon, \forall a, s, k, m \tag{44}$$

$$\pi_{is}^{km} \geq 0, \forall i, s, i \neq s, k \tag{45}$$

$$\Phi_{pi}\left(u, \pi, d, \mu, \phi, \eta\right) \geq 0, \forall i, s, i \neq s, k' \tag{46}$$

$$\pi_{is}^{km}\Phi_{pi}\left(u, \pi, d, \mu, \phi, \eta\right) \leq \epsilon, \forall k, m, i, s \tag{47}$$

$$d_{is}^{km} \geq 0, \forall k, m, i, s \tag{48}$$

$$\Phi_d\left(u,\pi,d,\mu,\phi,\eta\right) \geq 0, \forall k,m,i,s \tag{49}$$

$$d_{is}^{km}\Phi_d\left(u,\pi,d,\mu,\phi,\eta\right) \leq \epsilon, \forall k,m,i,s \tag{50}$$

$$\mu_{is}^{m} \geq 0, \forall i,s,m \tag{51}$$

$$\Phi_{mu}\left(u,\pi,d,\mu,\phi,\eta\right) \geq 0, \forall i,s,m \tag{52}$$

$$\mu_{is}^{m}\Phi_{mu}\left(u,\pi,d,\mu,\phi,\eta\right) \leq \epsilon, \forall i,s,m \tag{53}$$

$$\phi \geq 0 \tag{54}$$

$$\Phi_\phi\left(u,\pi,d,\mu,\phi,\eta\right) \geq 0 \tag{55}$$

$$\phi\Phi_\phi\left(u,\pi,d,\mu,\phi,\eta\right) \leq \epsilon, \forall i,s,m \tag{56}$$

$$\eta \geq 0 \tag{57}$$

$$\Phi_\eta\left(u,\pi,d,\mu,\phi,\eta\right) \geq 0, \forall i,s,m \tag{58}$$

$$\eta\Phi_\eta\left(u,\pi,d,\mu,\phi,\eta\right) \leq \epsilon, \forall i,s,m \tag{59}$$

$$\sum_m dd_{is}^{m} = Q_{is}, \forall i,s \tag{60}$$

Relaxed model with incentives:

$$\text{maximize} \sum_{k=1}^{T}\left[\gamma.\exp\left(\beta_0+\beta_1 k+\beta_2 k^2\right)\sum_{i}^{I} v_{is}^{k}\right] + \theta_1 \sum_{k<\bar{k}_0-\sigma}\sum_{i}^{I} v_{is}^{k} + \theta_2 \sum_{k>\bar{k}_0+\sigma}\sum_{i}^{I} v_{is}^{k} \tag{61}$$

subject to Eqs. 42–60

Relaxed System Social Optimum (TSO)

$$\text{maximize} \sum_{k=1}^{T}\left[\gamma \exp\left(\beta_0+\beta_1 k+\beta_2 k^2\right)\sum_{i}^{I} v_{is}^{k}\right] - \sum_{a,k}\tau_a^k(u)u \tag{62}$$

subject to Eqs. 42–60

All the problems above can be solved using well-established NLP solvers. However, as it was shown in Ban et al. (2010), they cannot be directly applied as the indicator parameters λ_1, λ_2, and λ_3 in Eqs. (14), (15), and (16) need to be fixed. The next subsection describes the approach to address this issue.

4.2 Algorithm Steps

The solution method is the iterative algorithm developed by Ban and Liu (2009). It solves the single level problem relaxed version of the problems by a certain reduction in the factor ϵ at each iteration, controlled by a update factor. There are four parameters that need to be determined before applying the algorithm: the initial relaxation parameter ϵ_0, the final relaxation parameter ϵ_f, the iteration limit *IL*, and the update factor ς. The selection of the four parameters represents certain trade-off between solution efficiency and quality (i.e., whether the problem can be solved or not). More details on how to select the four parameters can be found in Ban et al. (2010). In the following, the main steps of the relaxation solution algorithm are summarized.

Step 1: Initialization

Choose an initial relaxation parameter $\epsilon_0 > 0$. Set the iteration limit IL, update factor $0 < \varsigma < 1$, and $\kappa = 0$

Step 2: Major iteration

Step 2.1. Solve the current relaxed single level NLP (Relaxed Staggered Work Hour Problem) using the solution from last iteration as the starting point. Use ϵ_κ as the relaxation parameter in Equations (7.34–7.38).

Step 2.2. Update and move. If $\varsigma \leq IL$, set $\epsilon_{\kappa+1} = \varsigma\epsilon_\kappa$, $m = m + 1$, and go to Step 2.1; otherwise, go to Step 3.

Step 3. Final solve

Set the final relaxation parameter as ϵ_f which is a predefined value representing the desired solution accuracy. If it is successful, we obtain an optimal solution; otherwise, an approximate solution is achieved from the last run of Step 2.2.

Note that the algorithm only provides an iterative framework and can be readily implemented using standard NLP solvers to solve the relaxed single level problem in Step 2.1. Note also that the solution from previous iteration not only provides a starting point in Step 2.1 but it is also used to fix temporarily the three indicator functions λ_1, λ_2, and λ_3 so that the relaxed sub-problem can be represented as a regular NLP.

5 Application Example

The objective of this example is to perform an analysis of the feasibility of scheduling workers off the peak period, such as in staggered work hours, as well as on the effects on workers, firms, and society using the bi-level models presented in Section 4.

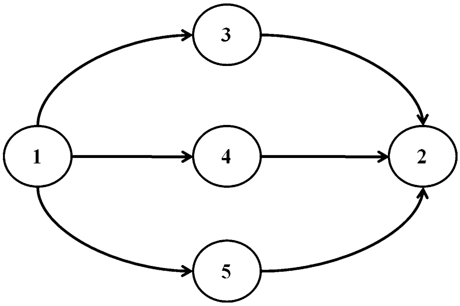
5.1 Network Description

The example uses a single OD pair network with three paths shown in Fig. 3. It consists of 4 nodes, with a single firm located at node 2, and its workers, a total of 2,000, located at node 1. This network provides 3 possible route choices: (1-3-2), (1-4-2), and (1-5-2), including a bottleneck in (3-2) due to the reduction in capacity. The details of the link data are shown in Table 2. Such values have been selected in order to match the average travel time in a US City (about 18–20 minutes) (Federal Highway Administration 2011). The link performance function used is given in Eq. 63.

$$\tau_a^k = \frac{L_a}{S_a}\left(1 + 0.15\frac{x_a^{k+1}}{L_a D_a}\right) \tag{63}$$

where L_a defines the length of the link, D_a the jam density, S_a the capacity, and x_a^{t+1} the total flow traversing link a at interval $t + 1$.

Fig. 3 Example network



5.2 Estimation of the Production Effects Parameters

The parameters of the model are based on the model estimated by Gutiérrez-i-Puigarnau and Ommeren (2012) and be can be found in Table 3. Such values are converted into travel time minutes in the objective functions of the moedls by assuming a value of time of €20/hr. The final production function is the slightly U-inverse function shown in Fig. 4). These values are later converted to US dollars for the analysis assuming a currency rate of $1.5 per €1.

5.3 Scenarios for Evaluation

In order to test the formulation and validate the effects of an staggered work hours scheme, two type of scenarios have been selected for evaluation. The first type of scenario assumes that workers have no additional costs different from the congestion costs (travel time and early/late penalty or $\alpha_1 \neq 0, \alpha_2 \neq 0$) which implies no opportunity costs ($\xi = 0$). The second type of scenario assumes both type of costs in workers disutility ($\alpha_1 \neq 0, \alpha_2 \neq 0, \xi \neq 0$). A description of the evaluated scenarios is provided below and summarized in Table 4.

Table 2 Network data for example 1

Link	Length (miles)	Capacity (vphpl)	No. lanes	Jam density (vpm)	Free flow speed (mph)
(1 – 3)	12.5	1,500	2	250	45
(1 – 4)	8	1,800	1	250	30
(1 – 5)	5	1,500	1	250	30
(3 – 2)	12.5	1,200	1	250	45
(4 – 2)	7	1,500	1	250	30
(5 – 4)	5	1,200	1	250	30

Table 3 Firm data for the logarithm, of wage from (Gutiérrez-i-Puigarnau and Ommeren 2012)

Category	Variable	Parameter
Morning start time	Start time (hours)	0.225
	Start time squared (hours)	−0.013
Schedule type	Flexible schedule	0.012
	Night work	−0.013
	Evening work	−0.036
Worker category	Self-employed	0.17
	White collar	0.144
	Civil servant	0.347
Firm size	less than 20	−0.165
	from 20 to 200	−0.107
	from 200 to 2,000	−0.047
	unknown	−0.006
Others	Daily working hours (in log)	0.422
	Weekly working hours (in log)	−0.252
	Household income (in log)	−0.037
	Household income unknown or zero (in log)	0.104

5.3.1 Scenario Type I: No Opportunity Costs

In this type of scenario, congestion becomes the major part of the cost (if not all) and the opportunity costs (non-work activity costs) are negligible. For this type of scenario, the following cases have been evaluated:

Base Case I: This case solves the general problem without incentive. It is assumed that the firm gives very little flexibility for arrival, 5 minutes earlier

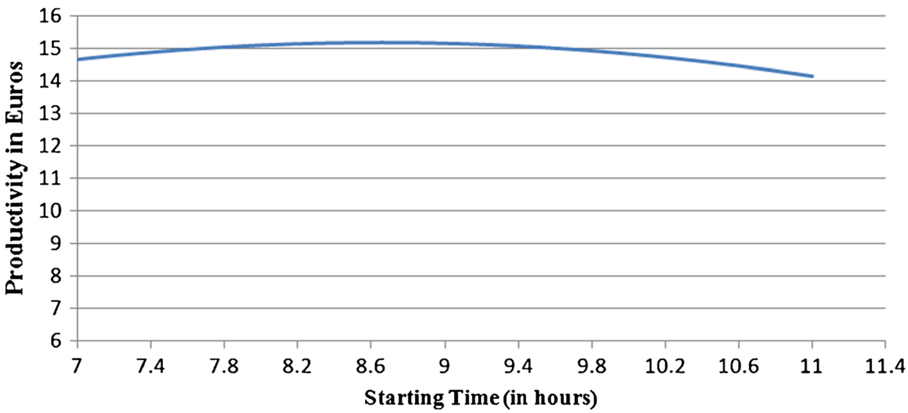


Fig. 4 Productivity function by starting time

Table 4 Parameters per scenario

Parameters	Scenario type 1 BC I	Case Ia	Case Ib	Scenario type 2 BC II	Case IIa	Case IIb
Interval length (Δ)	1 min	1 min	1 min	1 min	1 min	1 min
Max. demand*	400	400	400	400	400	400
Max. inflow**	75	75	75	75	75	75
Default start time	8:45 am	8:45 am	8:45 am	8:45 am	8:45 am	8:45 am
(Interval)	(80)	(80)	(80)	(80)	(80)	(80)
Quadratic opp. costs	No	No	No	Yes	Yes	Yes
Penalty (ξ)				0.5	0.5	0.5
Linear schedule delay	Yes	Yes	Yes	Yes	Yes	Yes
Early penalty (α_1)	2	2	2	2	2	2
Late penalty (α_2)	5	5	5	5	5	5
Tolerance in min (σ)	±5	±20	±5	±5	±20	±5

*per interval

**per interval per lane

and later from the starting time (8:45 am). The early penalty α_1 is 2 and the late penalty α_2 is 5; implying that late arrivals are more penalized. This first case serves also as the Base Case scenario for comparison of the effects with the other scenarios.

Case Ia: This is the case of reassigning workers off the peak period by providing an incentive to the firm to stagger workers. The incentive parameters in this case is set to $\theta = 0.2$ which is given to firm whenever workers arrive before 8:40 am or after 8:50 am. The α parameters, and the firm's preferred arrival time are the same as in the base case. However, the tolerance is relaxed up to 20 minutes, in order to not penalize workers who are now shifted away from the 10-min peak period.

Case Ib: This case solves the social system optimum model with the parameters defined in the Base Case.

5.3.2 Scenario Type II: Opportunity Costs

The second type of scenario assumes that due to some intermediate activities, workers have large opportunity costs, and prefer the peak starting time or the original starting time. This can be the case of parents with children, whose activities (such as school or daycare) are designed to match the original work schedule, forcing parents to travel during peak hours. This scenario implies that the worker has a strong preference for the peak period and a deviation from that preferred time imposes large costs. To account for this scenario, parameter ξ is set to 0.5. With this addition, the same scenarios analyzed in Scenario type I are evaluated, and numbered as Base Case II, Case IIa, and Case IIb respectively. The parameters used in each of these scenarios are shown in Table 4.

5.4 Results

5.4.1 Arrival Patterns

Scenario Type I As the Base Case scenario solves the problem without any enforcement or incentive to reschedule workers, it can be expected that given that the firm acts as the leader, it will produce a more concentration of arrivals around the solution that maximizes firm's utility. In this case, the concentration is around 8:43 am (Fig. 5). Despite having flexibility in arriving 5 minutes earlier or later, the equilibrium solution for workers show a high peak between 8:40 am and 8:45 am with almost no flow arriving after 8:45 am. This is expected as the penalty for late arrivals is 2.5 times less than the penalty for early arrivals. The social system optimum (referred as SSO or Case Ib) shows a more spread arrival pattern. In this case, the workers are also able to arrive late increasing the arrival interval from (8:26 am – 8:45 am) to (8:27 am – 8:50 am). However, Case Ib is not efficient in terms of arrivals when compared with Case Ia in which the firm receives a large incentive to stagger workers. In Case Ia, the period of arrivals extends up to 8:14 am as the earliest arrival and 8:50 am as the latest. Again it can be observed that early arrivals are still preferred than late arrivals. In addition, it can be noticed that the incentive scenario is strong enough to obtain a pattern with constant arrivals almost equal to the maximum inflow (75).

Scenario Type II Including opportunity costs increases the intensity of the peak-period in the base case (see Fig. 6) because now both firms and workers have

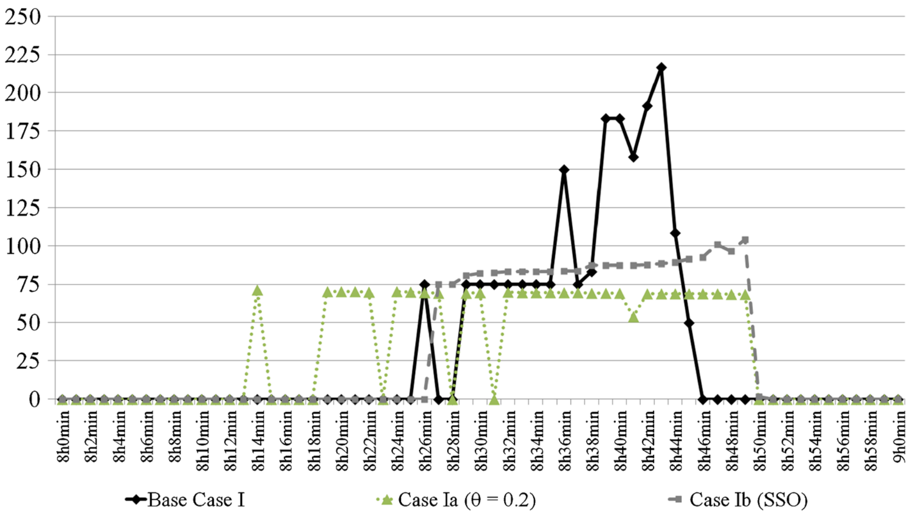


Fig. 5 Arrival profiles for scenario type I

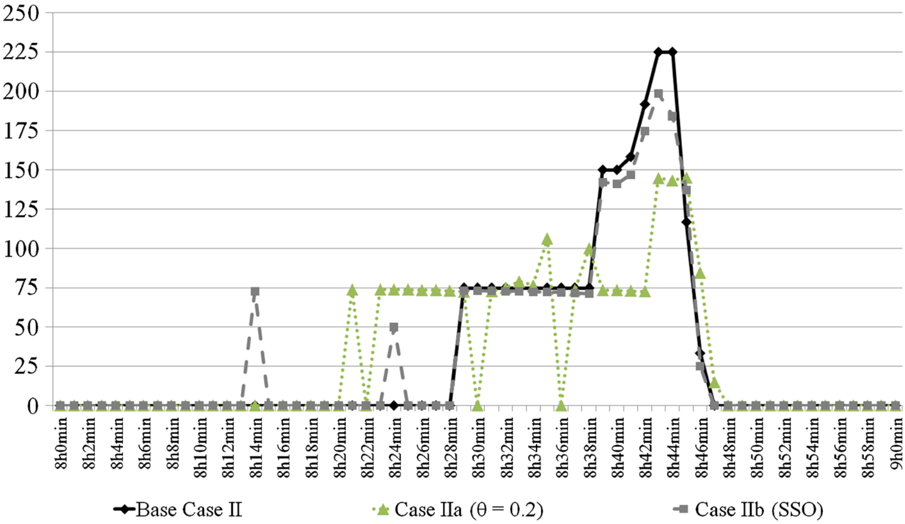


Fig. 6 Arrival profiles for scenario type II

preferences for the peak hour. In the case of the workers there are still some workers that compensate this increase in the opportunity costs with travel time savings and arriving before the peak-period. The provision of the incentive spreads the arrival pattern in a similar fashion as in Scenario I but will still maintain a slight concentration between 8:40 am and 8:50 am. On the other hand the social system optimum (SSO or Case IIb), by accounting now for workers preferences, includes a significant arrival between 8:43 am and 8:46 am following a similar pattern as in the base case.

5.4.2 System Performance

The system performance is evaluated in terms of the total system travel time and total system travel time costs for each type of scenario. For the travel time costs, a value of time of $30/hr has been used.

Scenario Type I The implications of the obtained arrival profiles in the total system travel time (TSTT) are summarized in Table 5. In the base case, the total travel time obtained was about 50,032 minutes, which represents an average travel time per traveler of 25.02 minutes. The incentive scenario (Case Ia) reduces the total traffic congestion for the morning commuting by 6,373 minutes which represent a reduction of 12.7 percent. When the social system optimum (Case Ib) is implemented, it reduces the traffic, however, the firm production effect is strong enough to maintain certain concentration reducing the benefits by 5,334.14 minutes or 10.6 percent. If a value of time of $30/hour is used, this represents savings during the morning commute of $3,186.66 and $2,667.07 for the incentive case and the social system optimum respectively. The average travel time for a worker has slightly been reduced

Table 5 Network effects in scenario I: no opportunity costs (demand = 2,000; $\xi = 0$)

		Base case (BC)	Case Ia ($\theta = \$0.2$)	Case Ib (SSO)
Networkwide	TSTT (in min)	50,332.553	43,659.24	44,698.414
Effects (morning	Savings w.r.t. BC (min)		(6,373.31)	(5,334.14)
commute)	TTST (in $)	24,668.28	21,829.62	23,349.21
	Savings w.r.t. BC (in $)		(3,186.66)	(2,667.07)
Total arrivals	From 8:00 to 8:15 am	–	66.39	–
	From 8:16 to 8:30 am	225.00	356.88	313.23
	From 8:31 to 8:45 am	1,775.00	1,197.66	1,290.50
	From 8:46 to 9:00 am	–	379.08	395.89

(TSTT = Total system travel time)

to 26.1 minutes in Case Ib and to 22.62 minutes in Case Ia or $4.69 per trip in Case Ib and $6.41 per trip in Case Ia.

Scenario Type II In scenario II type, the concentration around a peak period has implied higher transportation travel times with respect to scenario type I base case (about 1.18 percent). When comparing Case IIa and IIb with the base case, it can be noticed that Case IIa, when incentives are provided, spreads the arrivals and maintains a small peak between 8:43 am and 8:46 am (Table 6). This distribution of arrivals reduce the congestion by 4,627.14 minutes (9.1 percent) or $2,313.57. The social system optimum does not show such a flat distribution as in Scenario I because now workers have a preference for the regular starting time; the congestion is slightly reduced by $272.07 (544.14 minutes). These results are summarize in Table 6

5.4.3 Firm and Workers Performance

Firm benefits have been obtained directly from the objective function and converted into money by considering a value of time of $30/hour and 8 hours of work. Workers

Table 6 Network efects for scenario II: opportunity costs (demand = 2,000; $\xi = 0.5$)

		Base case (BC)	Case IIa ($\theta = \$0.2$)	Case IIb: (SSO)
Networkwide	TSTT (in min)	50,242.139	45,615.00	49,698.00
Effects (morning	Savings w.r.t. BC (in min)		(4,627.14)	(544.14)
commute)	TSTT (in $)	25,121.07	22,807.50	24,849.00
	Savings w.r.t. BC (in $)		(2,313.57)	(272.07)
Total arrivals	From 8:00 to 8:15 am	–	–	73.03
	From 8:16 to 8:30 am	150	589.71	197.00
	From 8:31 to 8:45 am	1,816.67	1,310.69	1,704.87
	From 8:46 to 9:00 am	33.33	99.60	25.11

(TSTT = Total system travel time)

have been evaluated using the disutility of travel which is only represented by the travel time and schedule delay in scenario type I and travel time and schedule delay plus the opportunity costs in scenario type II. Finally, the evaluation of the benefits to society includes the benefits of the firm (firms productivity per day) minus the costs imposed in society, which in this case includes the system travel time.

Scenario Type I: No Opportunity Costs The results indicate that as more workers are spread across starting times, the company reduces its benefits. The reductions are $1,906.68 when the incentive is provided and $2,120.83 for the social system optimum case. For workers, the equilibrium disutilities of travel (including early/late penalty) show savings of $6.41 per worker in the incentive case and $4.69 for the social system optimum (see Table 7). This implies that those workers without commitments in their arrival time can benefit if they negotiate their arrival time with the firm. This benefit is not only because of their travel time savings but also because of their reduction of the costs of the schedule delay associated with their work trip. These benefits at the end transfer to society.

In terms of the benefits to society, the social system optimum has been solved seeking to reduce the total system travel time and to maximize the productivity of the firm. This is attained with a more spread arrival than the base case but not as spread as in the incentive case. This is again because the firm production effects prevent a better solution in terms of total system travel time. In other words, what is maximized is the net benefit to the society which in this case leads to a more concentrated arrival pattern compared with in the incentive scenario. (see the last row -Social Costs- in Table 7)

Scenario Type II: Opportunity Costs Under this scenario, the case with incentives again impacts firms' productivity reducing the productivity by $2,721.85 (see Table 8). As it can be expected, the social system optimum has little impact in the productivity (a reduction of $107.95 with respect to the base case) because its arrival

Table 7 Effects in firm, workers, and society in scenario I: no opportunity costs (demand = 2,000; $\xi = 0$)

		Base case (BC)	Case Ia ($\theta = \$0.2$)	Case Ib (SSO)
Firm	Total (Hour)	45,800.77	45,437.43*	45,535,67
Productivity ($)	Per Employee	22.90	22.52	22.77
	Total (Day)	366,406.16	364,499.48*	364,285.33
	Loss w.r.t. BC		(1,906.68)	(2,120.83)
User costs	Disutility of travel (in min)	35.440	22.615	26.062
	Disutility of travel (in $)	17.720	11.308	13.031
	Savings w.r.t. BC (in $)		(6.41)	(4.69)
Society	Social benefits($):			
	(Firm Prod. - TSTT)	341,389.88	341,669.86	341,936.12

(*After discounting the amount of incentive)

Table 8 Effects in firm, workers, and society in scenario II: opportunity costs (demand = 2,000; $\xi = 0.5$)

		Base case (BC)	Case IIa ($\theta = \$0.2$)	Case Ib: (SSO)
Firm	Total (Hour)	45,881.22	45,540.99*	45,867.23
Productivity ($)	Per employee	22.94	22.77	22.93
	Total (Day)	367,049.78	364,327.93*	366,941.83
	Loss w.r.t. BC		(2,721.85)	(107.95)
User costs	Disutility of travel (in min)	39.84	44.13	40.01
	Disutility of travel (in $)	19.92	22.06	20.003
	Savings w.r.t. BC (in $)		2.140	0.080
Society	Social benefits($):			
	(Firm Prod. - TSTT)	341,928.71	341,520.43	342,092.83

(*After discounting the amount of incentive)

pattern is similar to the base case pattern. These changes in the productivity affect the net benefits to society (social costs).

In the case of the incentive scenario the significant travel time reduction cannot offset the reduction in productivity imposed by the firm. Thus it require the social system optimum to be closer to the base case. In fact, by considering the disutilities of the workers, the minimum is found in the base case, as there both workers and the firm have preferences for the peak period. Therefore, imposing a solution like that attained in the incentive scenario increases the disutility mainly because of the increase in the opportunity costs. In the example, it imposes an extra $2.14 dollars. The consequences of this increase in the disutilities also affects the social efficiency, because now the social system optimum (SSO) imposes an extra $1 in workers (see Social Costs in Table 8).

5.5 Equilibrium Analysis

The equilibrium analysis uses a game theoretical approach to understand the phenomena and to show more clearly the gains and losses in the agents and society in general. From the analysis, it will become evident how firms benefit from an asymmetric interaction in spite of the benefits of having a portion of its workers assigned to off-peaks can provide to society. The equilibrium analysis evaluates first the most efficient solutions found for the society in terms of net benefits accounting for the total system travel time costs. In fact, the analysis can be used to evaluate whether staggered work hours, or alternative work schedules, is a feasible choice for the government.

5.5.1 General Setting

A more clear explanation of this phenomena can be done using a Game Theory payoff matrix. In this matrix we can show both agents: a firm and a worker deciding their strategies. For the analysis, the firm and the workers have only 3 possible options: to follow a fixed work schedule with tight constraints (FO) -represented by the base case in the cases analyzed earlier-, to relax such constraints and assign a group of workers arrivals to less congested periods represented by the case with incentives (SWH), or to follow the social system optimum solution imposed by a planner (SSO). Only the pure combinations (both choosing FO, SWH or SSO only) are feasible options because only one schedule can be chosen at a time (i.e., if the schedule is set to the peak hour, workers cannot simply change their schedule to a less congested period without consent of the firm). The entries represent for the firm the daily productivity under the specific schedule, and for the worker represents its disutility of travel. Note that as payoffs we are assuming that the firm evaluates the change in schedules and account for the effects of all workers not for each individual worker. Moreover, this not only affects the arrival but also determines the daily productivity, which is the main reason why the daily productivity is included as payoff for the firm. On the workers side, if they do not have negotiation power, they negotiate independently so the payoff accounts for each individual worker payoff. Also it is assumed that each of them are playing pure strategies.

5.5.2 Scenario Type I: No Opportunity Costs

Consider the first scenario (workers without opportunity costs), as shown in Table 9. The best strategy for the worker is to choose SWH which is the solution attained if both the firms and workers decide to accept the incentive and stagger a significant 43.46 percent of the workers. By choosing this strategy, each worker saves \$6.41. However the firm, which evaluates the impact of choosing SWH (without including the incentive), is worse off by \$ 1,906.68. This is basically because the firm evaluates the effects of such strategy considering the overall effects and not individually considering the marginal loss if they move one employee to the SWH schedule. The same happens if they follow the planner's proposed schedule. The amount of loss in the firm offsets largely the amount of the benefit of each user. Thus the Nash

Table 9 Payoff matrix of scenario type I: no opportunity costs

Firm	Worker		
	FO (Base case) % stagg.: 11.25 %	SWH (Incentive) % stagg.: 43.46 %	SSO (Planner) % stagg.: 35.46 %
FO (BC/11.25 %)	(366,406.16, 17.52)*		
SWH (Incentive/43.46 %)		(364,499.48, 11.31)	
SSO (Planner/35.46 %)			(364,285.33, 13.031)

(*Nash equilibrium)

Equilibrium solution for the work schedule in Scenario I is FO: maintain a fixed single schedule at the peak period.

However, this solution might be misleading because neither the firm nor the workers are completely aware of the actual costs that are imposed to society. If they are left to interact with such asymmetry, the society becomes inefficient because on the contrary to how these two agents interact, society looks at the total costs and total benefits. In other words, society considers the total system travel time not just the individual perception of travel time saved by a worker. From a planner's perspective, the best solution would be to shift 35.46 percent of workers in which the trade-off between total travel time savings and production effects leads to a more efficient solution (see Table 10 after the incentive is discounted from the incentive scenario). Thus the intervention provides a more efficient solution than simply leaving the firm and workers negotiate by themselves. The optimal incentive to achieve the social system optimum would be equal to the amount of the loss incurred by the firm $2,120.83, which is covered by the travel time savings ($2,667.07).

5.5.3 Scenario Type II: Opportunity Costs

Now consider the case in which workers also have preference for arriving during the peak-period. The selection of strategies also leave the Nash Equilibrium in FO (see Table 11) because the firm incurs in a loss by changing to any other schedule, and the worker increases her cost if they shift to SWH and SSO.

Note that again this Nash Equilibrium solution produces traffic congestion and it is not socially beneficial as the total system travel time achieved is larger than those that can be obtained rescheduling workers. The preference from the worker's side and the production effects from the firm's side collaborate in choosing the most congested scenario. This congested scenario from the planner's perspective is not efficient because considering the travel time savings, society can be better either staggering using the incentive or leaving the planner to set up the schedules (see Table 12). In this case, the optimal incentive is $272.87 which includes both the loss of the firm ($107.95) but also the increase in the cost by the workers ($0.08/worker) almost completely covered by the travel time savings ($272.07)

Table 10 Summary of net benefits and costs in scenario type I: no opportunity costs

	Worker's costs	Total travel time cost (morning commute)	Firm's productivity (daily)	Benefits to society (daily)
FO	17.52	50,332.553	366,406.16	341,389.88
SWH	11.31	43,659.24	364,499.48*	341,669.86
SSO	13.03	44,698.414	364,285.33	341,936.12

Note: FO = Firm optimal/Base case, SWH = Staggered with incentive, SSO = Social system optimum

*After discounting the incentive

Table 11 Payoff matrix of scenario II: opportunity costs

Firm	Worker		
	FO (Base case) % stagg.: 9.17 %	SWH (Incentive) % stagg.: 34.47 %	SSO (Planner) % stagg.: 14.76 %
FO (Base Case/9.17 %)	(367,049.78, 19.92)*		
SWH (Incentive/34.47 %)		(364,327.93, 22.06)	
SSO (Planner/14.76 %)			(366,941.83, 20.003)

(*Nash equilibrium)

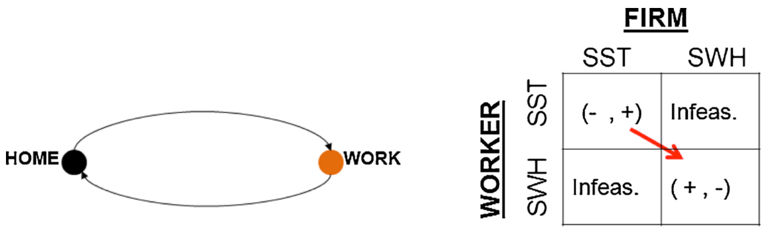
5.6 Summary of the Analysis

To summarize, with the previous analysis, two general situations can be identified, each one based on the type of scenario faced by the worker. In the first type of Scenario, it is assumed that since the worker is affected mainly by congestion cost, they prefer to arrive in an off-peak period. Therefore, a minus "-" sign can be assigned to the case of starting at the peak period choice and a plus "+" sign for the off-peak case. However, the firm still prefers their workers to arrive at the peak period given that productivity is lost with an off-peak arrival, thus they originally have a "+" for starting at the peak period and "-" for off-peak arrival. In other words, if firms were asked to reschedule workers in off-peak periods voluntarily, the firm will lose and the worker will win. This case is a "Battle of Sexes game", for which the equilibrium solution is to start at the peak period since the firm is the dominate agent in this game (Fig. 7a). However, this solution is not socially optimal. Thus, if we need to move the solution to the socially optimal equilibrium, the firm needs to be compensated so both agents win. This situation is similar to the carrier-receiver game for Off-Hour Deliveries (OHD) (Holguín-Veras 2008) where the receiver plays a dominant

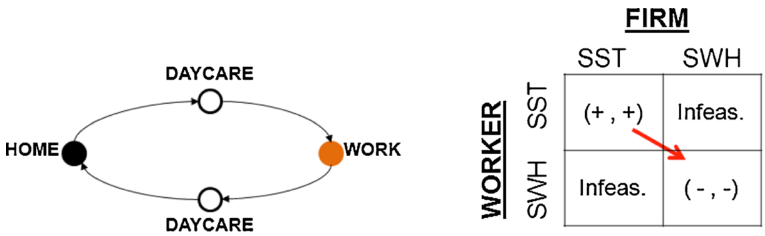
Table 12 Summary of net benefits and costs in scenario type II: opportunity costs

	Worker's costs	Total travel time cost (morning commute)	Firm's productivity (daily)	Benefits to society (daily)
FO	19.92	50,242.139	367,0.49.78	341,928.71
SWH	22.06	45,615.00	364,327.93*	341,520.43
SSO	20.00	49,689.00	366,941.83	342,092.83

Note: FO = Firm optimal/Base case, SWH = Staggered with incentive, SSO = Social system optimum
*After discounting the incentive



(a) Scenario 1: "Battle of Sexes"



(b) Scenario 2: "Cooperation"

Fig. 7 Summary scenarios

role who will be worse off if OHD is implemented, while the carrier will be usually better off.

In the second type of scenario, we observe that, given that the worker has some non-work activities before or after work, they are tied to the original work schedule. In this situation what it can be observed is that both the firm and the workers benefit from starting at the peak period which is a Nash equilibrium solution, called Cooperative Game (Fig. 7). The solution in both cases will necessarily reach a congested solution which is not socially optimal. Therefore under this type of situation both agents need to be compensated if a more socially beneficial solution needs to be achieved.

To summarize, the relaxation of work arrivals and the assignment to off-peak periods provide benefit to the overall society because it results in less congestion during the peak hour, but may hurt the firm or even the workers. It requires the intervention of a third party, meaning that the government should provide financial incentives to make it feasible and socially efficient. The incentives are to compensate the loss of the firm and, in certain cases, the possible loss in the workers. However, it looks from the example that it is more appropriate to target workers that have no opportunity costs imposed by other activities (i.e., parents with children).

6 Conclusions

There have been several attempts to reduce congestion during the peak hours. The most used approach in recent times has been congestion pricing. In spite of several efforts, congestion pricing has not been a solution for relieving congestion at least in the peak period. The failure in this mechanism is that it does not account for the nature of the work trips which are the majority of the traffic during such period. Work trips are usually constrained by inflexible or tight work schedules which makes difficult, if not impossible, for workers to shift to less congested moments. The shift to less congested period can be achieved with alternative work schedules in his multiple variants. Such strategies have been implemented since early 1950's. But despite initial success, it did not sustain over time.

In this paper, bi-level models are used to study the rescheduling of work activities. The models provide insights regarding why such strategy has not been widely adopted. In the development of the model, the behavior of the firm based on a production function and the behavior of workers based on the disutility of travel was analyzed. These behaviors were translated into a mathematical representation. The novelty of the approach is that, besides modeling workers and firm behaviors, it takes into account the actual interactions between workers and the firm, as well as it accounts for the network effects. The interaction is represented in a bi-level structure that captures the asymmetric distribution of power that benefits the firm, producing a market failure that does not benefit society as a whole.

Through an example, it has been shown that without intervention the firm's private utility imposes the work schedule which, in cases when the workers does not have opportunity costs (e.g., single workers or parents without children), makes them worse off. But in cases when the workers have opportunity costs both firms and workers collaborate and end up choosing the most congested scenario as the concentration of trips during the peak period is more beneficial for both agents. The two cases analyzed in each scenario resembles the "Battle of Sexes" and a "Cooperative Scheme." In the first case, workers are tied to firms decisions when they do not have preference about starting times. And the second case arises when workers have additional costs besides the commuting costs, resulting in a game in which both the firm and workers lose from staggered work hours.

In both cases, leaving the market without intervention imposes higher costs in society. Therefore, this study also includes the key role of a third agent acting as the system manager who provides necessary incentives in order shift workers to alternative work schedules. The incentive seeks to compensate the firm for the additional costs imposed by the new schedule. When the firm is affected by small production effects, it is easier for the planner to reach a more efficient solution and to finance the incentive with the travel time savings. Therefore, the incentive may turn out to be a critical component in making any kind of alternative work schedule policy practically feasible.

The paper has also shown how to achieve the solution that maximizes welfare. That is, one that accounts for the transportation network users (workers) as well as the firm. In such cases, a transfer from the congestion savings is expected and should offset the production effects on the firms. The optimal values for incentives to achieve these solutions are basically the change in the loss of productivity in the firm. From another perspective, a congestion tax may also be implemented by charging the firm with and increase in taxes if staggered work hours is not implemented. The amount of the increase should be equal to the increase in congestion due to starting at the peak period.

The policy implication from our initial study in this paper is that, in order induce a shift of work trips to less congested periods (i.e., staggered work hours), a third party is necessary to get involved, mainly providing incentives to the firm to compensate losses due to the rescheduling of work. If workers do not have opportunity costs, i.e., do not have preference for starting at specific time or if travel time costs are the most relevant ones, the firm incentive is enough to produce the shift to off-peak periods. However, if the new schedules have sufficient impact on workers, part of this incentive (or extra-funds) has to be transferred to workers. This has important policy implications, because in such environments, where workers can demand additional wages if their schedules are seriously affected, rescheduling of work starting times would require large incentives to compensate both agents. Obviously this depends on how large the induced non-work rearrangement activities costs are, which is difficult to measure for each person and can vary across time. A model able to capture the heterogeneity among workers in terms of their preferences towards work starting times is needed in future research to conduct more comprehensive analysis on how this policy should be implemented.

Appendix A: Notation for the DUE Model

Sets

M Defines the workers time schedule as time m (group of workers or "class m")

K Time intervals of length Δ

Variables

d_{is}^{km} Flow departing from node i at time interval k for people under starting work time schedule m to destination s

π_{is}^{km} Equilibrium travel time for flow departing from node i at time interval k for people under starting work time schedule m to destination s

u_{as}^{km} Inflow of group of workers with starting time m for departing at the beginning of time interval k to link a to destination s

v_{as}^{km} Exit flow of group of workers with starting time m departing at the beginning of interval k from link a to destination s

μ_{is}^{m} Disutility value for flow of group of workers with starting time m' for flow departing from origin node i to destination s

dd_{is}^{m} Total departure flow of group of workers with starting time m from node i to s

Parameters

Q Total number of workers in the firm, which is the summation of the workers departing from any node i to destination s (firms location), $Q = \sum_{\forall i} Q_{is}$

θ Incentive per worker by a unit shift in his/her schedule

α_1 Early arrival disutility factor

α_2 Late arrival disutility factor

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