

## Relativistic Magnetic Reconnection in Kerr Spacetime

Felipe A. Asenjo<sup>1,\*</sup> and Luca Comisso<sup>2,†</sup>

<sup>1</sup>*Facultad de Ingeniería y Ciencias, Universidad Adolfo Ibáñez, Santiago 7941169, Chile*

<sup>2</sup>*Department of Astrophysical Sciences and Princeton Plasma Physics Laboratory, Princeton University, Princeton, New Jersey 08544, USA*

(Received 3 August 2016; revised manuscript received 24 October 2016; published 2 February 2017)

The magnetic reconnection process is analyzed for relativistic magnetohydrodynamical plasmas around rotating black holes. A simple generalization of the Sweet-Parker model is used as a first approximation to the problem. The reconnection rate, as well as other important properties of the reconnection layer, has been calculated taking into account the effect of spacetime curvature. Azimuthal and radial current sheet configurations in the equatorial plane of the black hole have been studied, and the case of small black hole rotation rate has been analyzed. For the azimuthal configuration, it is found that the black hole rotation decreases the reconnection rate. On the other hand, in the radial configuration, it is the gravitational force created by the black hole mass that decreases the reconnection rate. These results establish a fundamental interaction between gravity and magnetic reconnection in astrophysical contexts.

DOI: 10.1103/PhysRevLett.118.055101

The rapid conversion of magnetic energy into plasma particle energy through the process of magnetic reconnection is of paramount importance in many astrophysical processes [1]. Magnetospheric substorms, coronal mass ejections, and stellar and gamma-ray flares are just a few examples of phenomena in which magnetic reconnection is thought to play a crucial role.

In recent years, significant work has been undertaken to better understand magnetic reconnection in magnetically dominated environments, where relativistic effects become significant [2]. This has led to the generalization of the classical Sweet-Parker and Petschek reconnection models to the special relativistic regime [3,4]. Furthermore, many numerical campaigns have been devoted to the investigation of the reconnection rate [5–12] and the particle acceleration [13–20] in this regime. Relativistic reconnection has been found to be a very efficient mechanism of magnetic energy conversion and particle acceleration, making it a primary candidate to explain nonthermal emissions from pulsar wind nebulae, gamma-ray bursts, and active galactic nuclei.

While special relativistic effects on the reconnection process are starting to become clear, the effects related to the spacetime curvature are far less explored and a detailed understanding is lacking. General relativistic magnetohydrodynamic simulations have repeatedly shown the formation of reconnection layers in the proximity of black holes [21–24], where spacetime curvature effects can be important. However, the difficulties related to the spatial and temporal resolution of typical reconnection processes have been such to prevent their thorough study.

A step forward in the comprehension of magnetic reconnection in curved spacetime could be taken by studying simple generalizations of known theoretical models. It is

the purpose of this Letter to develop such a theoretical study considering the contribution of the gravitational field of a rotating black hole on the magnetic reconnection process. Ultimately, our goal is to determine how the reconnection rate and other properties of the reconnection layer are modified by spacetime curvature effects.

*Governing equations.*—We consider a plasma governed by the equations of general relativistic magnetohydrodynamics (GRMHD) [25,26], which are composed by the continuity equation  $\nabla_\nu(\rho U^\nu) = 0$ , the energy-momentum equation

$$\nabla_\nu(\mathfrak{h}U^\mu U^\nu) = -\nabla^\mu p + J^\nu F^\mu{}_\nu, \quad (1)$$

and the resistive Ohm's law

$$U^\nu F^\mu{}_\nu = \eta(J^\mu - \rho'_e U^\mu). \quad (2)$$

Here,  $\nabla_\nu$  denotes the covariant derivative associated to the connections of the curved spacetime, and  $U^\mu$  and  $J^\mu$  are the four-velocity and four-current density, respectively, while  $F^{\mu\nu}$  is the electromagnetic field tensor. Furthermore,  $\rho$ ,  $\mathfrak{h}$ , and  $p$  are the mass density, enthalpy density, and pressure of the plasma, respectively. Finally,  $\eta$  is the electrical resistivity, and  $\rho'_e = -U^\nu J_\nu$  is the charge density observed by the local center-of-mass frame of the plasma. The description of the dynamics is then completed when the fluid equations are complemented by Maxwell's equations  $\nabla_\nu F^{\mu\nu} = J^\mu$  and  $\nabla_\nu F^{*\mu\nu} = 0$ , where  $F^{*\mu\nu}$  is the dual of the electromagnetic field tensor.

A very effective representation of these equations can be obtained by writing them in the 3 + 1 formalism [27]. In this case, the curvature effects are displayed explicitly in a set of vectorial GRMHD equations. In this formulation, the line element can be written

as  $ds^2 = g_{\mu\nu} dx^\mu dx^\nu = -\alpha^2 dt^2 + \sum_{i=1}^3 (h_i dx^i - \alpha \beta^i dt)^2$ , where  $\alpha = [h_0^2 + \sum_{i=1}^3 (h_i \omega_i)^2]^{1/2}$  is the lapse function and  $\beta^i = h_i \omega_i / \alpha$  is the shift vector, while  $h_0^2 = -g_{00}$ ,  $h_i^2 = g_{ii}$ , and  $h_i^2 \omega_i = -g_{i0} = -g_{0i}$  are the nonzero components of the metric. Notice that the shift vector can also be written as  $\beta^i = \pm \sqrt{\alpha^2 - h_0^2} / \alpha$ , depending on the direction of the black hole rotation. Using the 3 + 1 formalism, we can isolate the effects of the spacetime rotation, which is useful for studying plasmas around rotating compact objects. It is also particularly convenient to introduce a locally nonrotating frame, the so-called “zero-angular-momentum-observer” (ZAMO) frame [28], where the line element is  $ds^2 = -\hat{d}t^2 + \sum_{i=1}^3 (\hat{d}x^i)^2 = \eta_{\mu\nu} \hat{d}x^\mu \hat{d}x^\nu$ , with  $\hat{d}t = \alpha dt$  and  $\hat{d}x^i = h_i dx^i - \alpha \beta^i dt$  (here and in the following, quantities observed in the ZAMO frame are denoted with hats). For observers in the ZAMO frame, the spacetime is locally Minkowskian. Using this frame, any equation with the form of Eqs. (1) and (2) can be cast into the general form  $\nabla_\nu S^{\mu\nu} = H^\mu$ . For example,  $S^{\mu\nu} = \mathfrak{h} U^\mu U^\nu$  for the energy-momentum equation and  $S^{\mu\nu} = 0$  for the Ohm’s law, while the form of  $H^\mu$  can be deduced from Eqs. (1) and (2). This general equation can then be written in the ZAMO frame in the vectorial form as [29]

$$\begin{aligned} & \frac{1}{h_1 h_2 h_3} \sum_{j=1}^3 \frac{\partial}{\partial x^j} \left[ \frac{\alpha h_1 h_2 h_3}{h_j} (\hat{S}^{ij} + \beta^j \hat{S}^{i0}) \right] + \frac{\hat{S}^{00}}{h_i} \frac{\partial \alpha}{\partial x^i} \\ & - \alpha \sum_{j=1}^3 (G_{ij} \hat{S}^{ij} - G_{ji} \hat{S}^{jj} + \beta^j G_{ij} \hat{S}^{0i} - \beta^j G_{ji} \hat{S}^{0j}) \\ & + \sum_{j=1}^3 \sigma_{ji} \hat{S}^{0j} = \alpha \hat{H}^i, \end{aligned} \quad (3)$$

with  $i = 1, 2, 3$ , and where  $G_{ij} = -(1/h_i h_j) \partial h_i / \partial x^j$  and  $\sigma_{ij} = -(1/h_j) \partial(\alpha \beta^i) / \partial x^j$ . Vectors and tensors observed by the ZAMO frame are related to the covariant vectors and tensors as  $\hat{S}^{00} = \alpha^2 S^{00}$ ,  $\hat{S}^{0j} = \alpha h_j S^{0j} - \beta^j \hat{S}^{00}$ , and  $\hat{S}^{ij} = h_i h_j S^{ij} - \beta^i \hat{S}^{0j} - \beta^j \hat{S}^{0i} - \beta^i \beta^j \hat{S}^{00}$ . Analogously, Maxwell equations can also be written in the ZAMO frame (see Ref. [29]).

Since we are interested in analyzing magnetic reconnection around black holes, we consider the spacetime  $(x^0, x^1, x^2, x^3) = (t, r, \theta, \phi)$  given by the Kerr metric [30], for which  $h_0 = (1 - 2r_g r / \Sigma)^{1/2}$ ,  $h_1 = (\Sigma / \Delta)^{1/2}$ ,  $h_2 = \Sigma^{1/2}$ ,  $h_3 = (A / \Sigma)^{1/2} \sin \theta$ ,  $\omega_1 = \omega_2 = 0$ , and  $\omega_3 = 2r_g^2 ar / \Sigma$ . Here,  $r_g = GM$  is the gravitational radius ( $G$  is the gravitational constant and  $M$  is the mass of the compact object), and  $a = J / J_{\max} \leq 1$  is the rotation parameter ( $J$  is the angular momentum and  $J_{\max} = GM^2$ ). Moreover,  $\Sigma = r^2 + (ar_g)^2 \cos^2 \theta$ ,  $\Delta = r^2 - 2r_g r + (ar_g)^2$ , and  $A = [r^2 + (ar_g)^2]^2 - \Delta (ar_g)^2 \sin^2 \theta$ . Finally,  $\alpha = (\Delta \Sigma / A)^{1/2}$  and  $\beta^j = \beta^\phi \delta^{j\phi}$ , where  $\beta^\phi = h_3 \omega_3 / \alpha$  is a measurement of the rotation of this spacetime. Note that in Kerr spacetime we have  $\beta^j G_{ij} \equiv 0$ .

In our analysis, we adopt a Sweet-Parker-like approach [1]; i.e., we look at magnetic reconnection under quasi-stationary conditions ( $\partial_t \approx 0$ ) within narrow ( $\delta \ll L$ ) quasi-two-dimensional current sheets. Quasistationarity is generally satisfied not only in the steady state but also at the time of maximum reconnection rate. Current sheets where magnetic diffusion takes place can form in different locations around black holes. Here, we consider two paradigmatic cases with current sheets in the equatorial plane ( $\theta = \pi/2$ ) of the massive body. We assume that these current sheets are in a stable orbit around the Kerr black hole [31].

*Reconnection layer in the azimuthal direction.*—We first consider a current sheet in the azimuthal direction, as shown in Fig. 1. The magnetic field just upstream of this sheet is in the  $\phi$  direction and has magnitude  $\hat{B}_0$ . In the diffusion region,  $\hat{v}^r$  vanishes at the neutral line (where  $\hat{B}^\phi = 0$ ). Similarly, at the neutral line  $\hat{J}^\phi \approx 0 \approx \hat{J}^r$ , implying that  $\rho'_e$  vanishes. We assume  $\hat{v}^\theta \approx 0$ ,  $\hat{B}^\theta \approx 0$ , and that spatial variations of the fields with respect to  $\theta$  (latitude) are negligible, i.e.,  $\partial_\theta \approx 0$ . The same configuration has been adopted by Koide and Arai [32] to examine the possibility of energy extraction from a rotating black hole. As in their work, we assume that the plasma in this configuration rotates in a circular orbit with Keplerian velocity.

We can calculate the outflow velocity from the diffusion region,  $\hat{v}_o$ , from the spatial  $\phi$  component of the energy-momentum equation (1). This equation can be written in the ZAMO frame by using Eq. (3). Then, evaluating along the neutral line we get

$$\frac{1}{\alpha h_1 h_2} \frac{\partial}{\partial \phi} [\alpha h_1 h_2 \mathfrak{h} \hat{\gamma}^2 \hat{v}^\phi (\hat{v}^\phi + \beta^\phi)] = -h_3 \hat{J}^\theta \hat{B}_r - \frac{\partial p}{\partial \phi}, \quad (4)$$

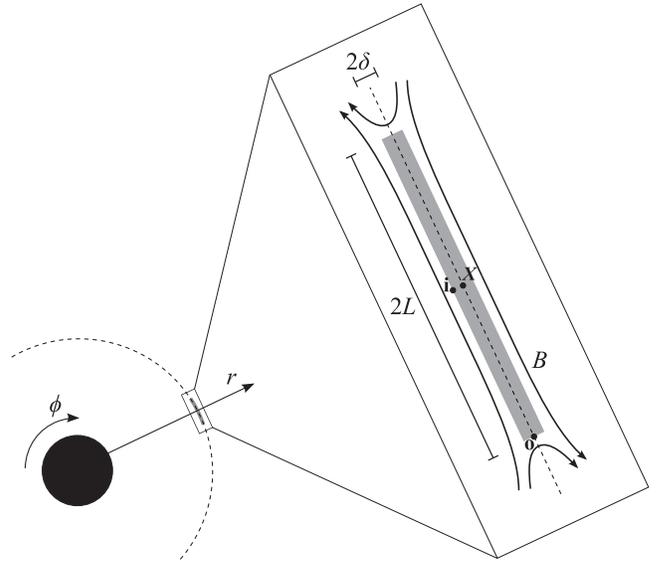


FIG. 1. Sketch of a reconnection layer in the azimuthal direction. The rotating black hole is represented by the black circle, while the magnetic diffusion region is marked by the shaded area.

where  $\hat{\gamma} = (1 - \hat{v}^2)^{-1/2}$  is the Lorentz factor. The outflow velocity  $\hat{v}_o$  can be found by integrating Eq. (4) from  $\phi_X$  ( $X$  point) to  $\phi_o$  (outflow point). For this calculation, we assume that the rotation of the black hole is slow compared to the outflow velocity,  $\hat{v}_o \gg \beta^\phi$ . This assumption is justified *a posteriori*. Note that pressure balance across the layer gives  $p_X \approx \hat{B}_0^2/2$ , while magnetic flux conservation yields

$$\hat{B}_r|_o \approx \left( \frac{r h_1}{h_3} \right) \Big|_o \frac{\delta}{L} \hat{B}_0, \quad (5)$$

where the symbol  $|_o$  indicates that the relevant quantities are evaluated at the outflow point. Strictly speaking, the magnetic flux in the diffusion region is not fully conserved, but Eq. (5) is an estimated approximation for the reconnected magnetic field. In principle, one can derive the Sweet-Parker reconnection rate without using this condition. Furthermore, from Maxwell's equations we can evaluate

$$\hat{J}^\theta|_o \approx \frac{1}{h_1} \Big|_o \frac{\hat{B}_0}{\delta}. \quad (6)$$

Thereby, by using the above assumptions and Eqs. (5) and (6), we can finally integrate Eq. (4) to obtain

$$\hat{\gamma}_o \hat{v}_o \approx \frac{1}{2}. \quad (7)$$

Here, we have considered a relativistically hot plasma, i.e.,  $\mathfrak{h} \approx 4p$  [33]. From Eq. (7), we have that the Lorentz factor of the plasma outflow is  $\hat{\gamma}_o \approx \mathcal{O}(1)$ , which implies a mildly relativistic outflow velocity  $\hat{v}_o \approx \mathcal{O}(1)$ , as in the flat spacetime limit [3,4]. Note that  $\hat{v}_o \gg \beta^\phi \approx 2ar_g^2/r^2$  when the current sheet is far from the black hole.

We proceed further by seeking the solution for the inflow velocity  $\hat{v}_i$ , which is a measure of the reconnection rate, and the current sheet width  $\delta$ , which, by means of Eq. (5), also leads to the reconnected magnetic field  $\hat{B}_r|_o$ . For this purpose, it is useful to consider separately the inner region, where magnetic diffusion occurs, from the outer region, where the plasma moves with a transport velocity that preserves the magnetic connections between plasma elements [34–37]. In the flat spacetime limit,  $\partial_t \approx 0$  and  $\partial_\phi \approx 0$  imply that  $\hat{E}_\theta$  is uniform and can be used as a matching condition for these two regions. In a more general case,  $\hat{E}_\theta$  can change in the equatorial plane, but for our purpose we need only to consider  $\hat{E}_\theta$  at the  $X$  point and the inflow point. From Maxwell's equations we have  $\partial_r(\alpha h_2 \hat{E}_\theta) = 0$  along the inflow line passing through the  $X$  point, implying that  $\hat{E}_\theta|_i \approx \hat{E}_\theta|_X$  if the current sheet width  $\delta$  is small, as can be seen *a posteriori*.

Inside the current sheet, the  $\theta$  component of Eq. (2) written in the ZAMO frame reduces to

$$\hat{\gamma} \hat{E}_\theta + \hat{\gamma} \hat{v}^\phi \hat{B}_r = \eta \hat{J}^\theta. \quad (8)$$

When evaluated at the  $X$  point, this equation simply is  $\hat{E}_\theta|_X = \eta \hat{J}^\theta$ . On the other hand, outside the current sheet the

plasma response to the electromagnetic field is well described by the ideal Ohm's law. Therefore, the rhs of Eq. (2) can be neglected in this outer region. This implies that the  $\theta$  component of the electric field at the inflow point becomes just  $\hat{E}_\theta|_i \approx \hat{v}_i \hat{B}_0$ .

The final step requires the estimation of  $\delta$ , which can be achieved from flux conservation. The inflow flux  $\partial_r(\alpha h_2 h_3 \hat{\gamma}_i \hat{\rho} \hat{v}_i)/(h_1 h_2 h_3) \approx \alpha \hat{\rho} \hat{\gamma}_i \hat{v}_i/(h_1 \delta)$  must balance the outflow flux  $\partial_\phi(\alpha h_1 h_2 \hat{\gamma}_o \hat{\rho} \hat{v}_o)/(h_1 h_2 h_3) \approx \alpha r \hat{\rho} \hat{\gamma}_o \hat{v}_o/(h_3 L)$ . This leads us to find

$$\delta \approx \left( \frac{h_3}{h_1 r} \right) \Big|_o \frac{\hat{\gamma}_i \hat{v}_i}{\hat{\gamma}_o \hat{v}_o} L, \quad (9)$$

and, finally, the inflow velocity (reconnection rate)

$$\hat{v}_i \approx S^{-1/2} \left( \frac{r}{h_3} \Big|_o \right)^{1/2}. \quad (10)$$

Here,  $S \equiv L/\eta$  is the relativistic Lundquist number ( $c = 1$  in our units), which compares the dynamical and resistive diffusion time scales of the plasma. We have considered  $\hat{\gamma}_i$  of the order of unity as  $S \gg 1$  for astrophysical plasmas. The spacetime curvature leads to  $h_3/r|_o > 1$ , implying a lower reconnection rate with respect to the flat spacetime limit. This can be seen more explicitly in the scenario where the diffusion region is sufficiently far from the black hole, where the reconnection rate can be approximated as

$$\hat{v}_i \approx S^{-1/2} \left( 1 - \frac{a^2 r_g^2}{4r_o^2} \right), \quad (11)$$

with  $r_o = r|_o$ . We clearly see that the spacetime curvature effects lead to a decrease of the reconnection rate in this configuration. The responsible factor is the rotation of the black hole, while the curvature created by the black hole mass itself does not play an important role if the current sheet is in the azimuthal direction far from the black hole.

*Reconnection layer in the radial direction.*—We now consider a magnetic reconnection configuration in which a narrow current sheet is located in the radial direction, as shown in Fig. 2. The magnetic field just upstream of the current sheet is in the  $r$  direction and has magnitude  $\hat{B}_0$ . In the diffusion region, the  $\theta$  component of the velocity vanishes at the neutral line (where  $\hat{B}^r = 0$ ). We assume  $\hat{v}^\phi \approx 0 \approx \hat{B}^\phi$  and  $\partial_\phi \approx 0$ . This implies that  $\rho'_e$  vanishes. This configuration is particularly relevant for the split-monopole magnetic field geometry, where reconnection layers form in the radial direction [23].

In this case, the outflow velocity  $\hat{v}_o$  can be calculated by considering the spatial  $r$  component of the energy-momentum equation (1). Writing it in the ZAMO frame (3) and evaluating it along the neutral line, we get

$$\begin{aligned} \frac{\partial}{\partial r} [\alpha h_2 h_3 \mathfrak{h} \hat{\gamma}^2 (\hat{v}^r)^2] + h_2 h_3 \frac{\partial \alpha}{\partial r} \mathfrak{h} \rho^2 \\ = -\alpha h_2 h_3 \left( \frac{\partial p}{\partial r} + h_1 \hat{J}^\phi \hat{B}_\theta \right). \end{aligned} \quad (12)$$

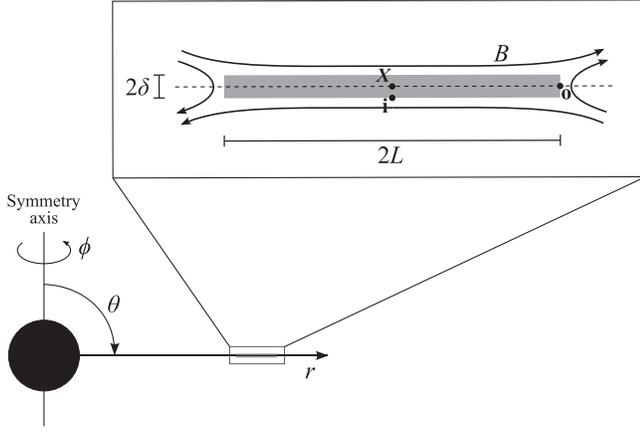


FIG. 2. Sketch of a reconnection layer in the radial direction. The rotating black hole is represented by the black circle, while the magnetic diffusion region is marked by the shaded area.

In the integration of this equation from  $r_X$  to  $r_o$  (in order to obtain the outflow velocity), we assume that the gravitational tidal field is negligible. This implies that the gravitational effects on different zones of that region, i.e., at  $r_X$  and  $r_o$ , are essentially the same. This also allows us to assume that the current sheet does not fall toward the black hole, and it can be considered in a rotational equilibrium. The above assumptions are no longer valid if the length of the current sheet [38] is comparable to the radial distance to the black hole or if the current sheet is too close to it. In that case, a more detailed approach must be used, which is left for future works. Here, we restrict ourselves to this simplified model in order to obtain the first approximated contribution of gravity to magnetic reconnection. Thus, at the integration level, the contribution of the spacetime curvature can be evaluated at the outflow point. The  $\phi$  component of the current density and the  $\theta$  component of the magnetic field at the outflow point can be estimated from Maxwell's equations and magnetic flux conservation. From them, we obtain

$$\hat{j}^\phi|_o \approx - \left( \frac{\partial_\theta(\alpha h_1 \hat{B}_r)}{\alpha h_1 h_2} \right) \Big|_o \approx - \frac{\hat{B}_\theta}{\delta} \quad (13)$$

and

$$\hat{B}_\theta|_o \approx \frac{1}{h_1} \Big|_o \frac{\delta}{L} \hat{B}_0, \quad (14)$$

where  $\hat{B}_0$  is in the radial direction. Then, considering a relativistically hot plasma and pressure balance across the layer,  $\hat{B}_0^2/2 \approx p_X$ , from the integration of Eq. (12) along  $r$  we find (at the same order)

$$\hat{\gamma}_o^2 \hat{v}_o^2 + L \frac{\partial \ln \alpha}{\partial r} \Big|_o \hat{\gamma}_o^2 \approx 1, \quad (15)$$

where Eqs. (13) and (14) have been used. Hence, the plasma outflow is characterized by a Lorentz factor and an outflow velocity given, respectively, by

$$\hat{\gamma}_o \approx \left( 1 + L \frac{\partial \ln \alpha}{\partial r} \Big|_o \right)^{-1/2}, \quad \hat{v}_o \approx \left( \frac{1}{2} - \frac{L}{2} \frac{\partial \ln \alpha}{\partial r} \Big|_o \right)^{1/2}. \quad (16)$$

In the flat spacetime limit ( $\alpha \rightarrow 1$ ), we recover again a mildly relativistic outflow velocity [3,4].

From Maxwell's equations [29], we have  $\partial_\theta(\alpha h_3 \hat{E}_\phi) = 0$  along the inflow line passing through the X point, implying that  $\hat{E}_\phi|_i \approx \hat{E}_\phi|_X$  for a small  $\delta$ . Therefore, we can match the electric field considering Ohm's law inside and outside the current sheet at the inflow point. Inside the current sheet, the  $\phi$  component of Eq. (2) written in the ZAMO formalism (3) becomes

$$\hat{\gamma} \hat{E}_\phi + \hat{\gamma} \hat{v}^r \hat{B}_\theta = \eta \hat{j}^\phi. \quad (17)$$

At the X point, this equation gives  $\hat{E}_\phi|_X = \eta \hat{j}^\phi$ . The  $\phi$  component of the electric field of this equation has to be matched with that coming from the ideal Ohm's law evaluated at the inflow point, i.e.,  $\hat{E}_\phi|_i \approx \hat{v}_i \hat{B}_0$ . This leads us to find the inflow velocity once the current sheet width is evaluated. From the balance between the energy inflow and outflow, we obtain

$$\delta \approx h_1|_o \frac{\hat{\gamma}_i \hat{v}_i}{\hat{\gamma}_o \hat{v}_o} L. \quad (18)$$

Therefore, the inflow velocity (reconnection rate) turns out to be

$$\hat{v}_i \approx \frac{1}{\sqrt{S h_1|_o}} \left[ \frac{1 - L \partial_r \ln \alpha|_o}{1 + L \partial_r \ln \alpha|_o} \right]^{1/4}, \quad (19)$$

where again  $\hat{\gamma}_i$  is of unity order because  $S \gg 1$ . Equation (19) contains the contribution of the gravitational field, from which we can clearly see that the spacetime curvature of the Kerr black hole modifies the reconnection rate. If we consider the ordering  $r_g \ll r_o$ , the reconnection rate from Eq. (19) can be approximated as

$$\hat{v}_i \approx S^{-1/2} \left[ 1 - \frac{r_g}{2r_o} + \frac{(2a^2 - 3)r_g^2}{8r_o^2} \right]. \quad (20)$$

This expression shows that the curvature created by the black hole mass leads to a decrease of the reconnection rate. On the contrary, the black hole rotation produces the opposite effect of increasing the reconnection rate. We observe, however, that these effects are small in the regime of validity of Eq. (20).

*Conclusions.*—The presented analysis has allowed us to calculate the reconnection rate and other important properties of the reconnection layer for two configurations in the equatorial plane of a rotating black hole. We have shown how the spacetime curvature modifies the magnetic reconnection process in comparison to the flat spacetime limit. The rotation of the black hole leads to a decrease of the reconnection rate if the reconnection layer is in the azimuthal configuration. On the other hand, it is the spacetime curvature due to the black hole mass that acts to decrease the reconnection rate if the current sheet is in the radial direction.

These results have been obtained for a plasma not close to the event horizon of the black hole and in a stable orbit [31]. We have also assumed a small rotation rate of the black hole. In future studies, these assumptions may be relaxed. We also observe that collisionless effects are important in plasmas around black holes, and, thereby, it is expected that they can also couple to gravity and affect the reconnection rate. A poloidal configuration could also be examined, and the black hole rotation is expected to modify the reconnection rate in a similar fashion to Eq. (11). This is relevant for exploring reconnection events in the corona and jet-launching regions of the black hole [21,40–44].

We finally observe that the potential formation of extremely elongated current sheets would result in slow reconnection rates. However, this situation is prevented by the occurrence of the plasmoid instability [39], which leads to the breakup of the current sheet in a fractal-like fashion [45] that ends when the smallest elementary current sheets are sufficiently thick to avoid the plasmoid instability. Our results can then be applied to these elementary current sheets, which are the actual locations where the magnetic energy is converted into plasma particle energy. Furthermore, our analysis can also be useful to better understand fast reconnection driven by turbulence [6,12,15], where the Sweet-Parker configuration occurs in several current sheets simultaneously due to the wandering of the magnetic field lines.

It is a pleasure to acknowledge fruitful discussions with Gabriele Brambilla, Luis Lehner, Russell Kulsrud, and Alexander Tchekhovskoy. F. A. A. thanks Fondecyt-Chile Grant No. 11140025. L. C. is grateful for the hospitality of the Universidad Adolfo Ibáñez, where part of this work was done.

---

\*felipe.asenjo@uai.cl

†lcomisso@princeton.edu

- [1] R. M. Kulsrud, *Plasma Physics for Astrophysics* (Princeton University, Princeton, NJ, 2005).
- [2] M. Hoshino and Y. Lyubarsky, *Space Sci. Rev.* **173**, 521 (2012).
- [3] Y. E. Lyubarsky, *Mon. Not. R. Astron. Soc.* **358**, 113 (2005).
- [4] L. Comisso and F. A. Asenjo, *Phys. Rev. Lett.* **113**, 045001 (2014).
- [5] S. Zenitani, M. Hesse, and A. Klimas, *Astrophys. J.* **696**, 1385 (2009).
- [6] G. Kowal, A. Lazarian, E. T. Vishniac, and K. Otmianowska-Mazur, *Astrophys. J.* **700**, 63 (2009).
- [7] S. Zenitani, M. Hesse, and A. Klimas, *Astrophys. J. Lett.* **716**, L214 (2010).
- [8] H. R. Takahashi, T. Kudoh, Y. Masada, and J. Matsumoto, *Astrophys. J. Lett.* **739**, L53 (2011).
- [9] N. Bessho and A. Bhattacharjee, *Astrophys. J.* **750**, 129 (2012).
- [10] M. Takamoto, *Astrophys. J. Lett.* **775**, 50 (2013).
- [11] Y.-H. Liu, F. Guo, W. Daughton, H. Li, and M. Hesse, *Phys. Rev. Lett.* **114**, 095002 (2015).
- [12] M. Takamoto, T. Inoue, and A. Lazarian, *Astrophys. J.* **815**, 16 (2015).
- [13] S. Zenitani and M. Hoshino, *Astrophys. J.* **670**, 702 (2007).
- [14] G. Kowal, E. M. de Gouveia Dal Pino, and A. Lazarian, *Astrophys. J.* **735**, 102 (2011).
- [15] G. Kowal, E. M. de Gouveia Dal Pino, and A. Lazarian, *Phys. Rev. Lett.* **108**, 241102 (2012).
- [16] B. Cerutti, D. A. Uzdensky, and M. C. Begelman, *Astrophys. J.* **746**, 148 (2012).
- [17] L. Sironi and A. Spitkovsky, *Astrophys. J. Lett.* **783**, L21 (2014).
- [18] F. Guo, H. Li, W. Daughton, and Y.-H. Liu, *Phys. Rev. Lett.* **113**, 155005 (2014).
- [19] F. Guo, Y.-H. Liu, W. Daughton, and H. Li, *Astrophys. J.* **806**, 167 (2015).
- [20] G. R. Werner, D. A. Uzdensky, B. Cerutti, K. Nalewajko, and M. C. Begelman, *Astrophys. J. Lett.* **816**, L8 (2016).
- [21] S. Koide, T. Kudoh, and K. Shibata, *Phys. Rev. D* **74**, 044005 (2006).
- [22] R. F. Penna, J. C. McKinney, R. Narayan, A. Tchekhovskoy, R. Shafee, and J. E. McClintock, *Mon. Not. R. Astron. Soc.* **408**, 752 (2010).
- [23] M. Lyutikov and J. C. McKinney, *Phys. Rev. D* **84**, 084019 (2011).
- [24] J. C. McKinney, A. Tchekhovskoy, and R. D. Blandford, *Mon. Not. R. Astron. Soc.* **423**, 3083 (2012).
- [25] A. Lichnerowicz, *Relativistic Hydrodynamics and Magnetohydrodynamics* (Benjamin, New York, 1967).
- [26] A. M. Anile, *Relativistic Fluids and Magneto-Fluids* (Cambridge University Press, Cambridge, England, 1989).
- [27] K. S. Thorne and D. A. Macdonald, *Mon. Not. R. Astron. Soc.* **198**, 339 (1982).
- [28] J. M. Bardeen, W. H. Press, and S. A. Teukolsky, *Astrophys. J.* **178**, 347 (1972).
- [29] S. Koide, *Astrophys. J.* **708**, 1459 (2010).
- [30] S. Weinberg, *Gravitation and Cosmology: Principles and Applications of the General Theory of Relativity* (Wiley, New York, 1972).
- [31] M. A. Abramowicz, A. Lanza, and M. J. Percival, *Astrophys. J.* **479**, 179 (1997).
- [32] S. Koide and K. Arai, *Astrophys. J.* **682**, 1124 (2008).
- [33] S. Chandrasekhar, *An Introduction to the Study of Stellar Structure* (Dover, New York, 1938).
- [34] W. A. Newcomb, *Ann. Phys. (N.Y.)* **3**, 347 (1958).
- [35] F. Pegoraro, *Europhys. Lett.* **99**, 35001 (2012).
- [36] F. A. Asenjo and L. Comisso, *Phys. Rev. Lett.* **114**, 115003 (2015).
- [37] F. A. Asenjo, L. Comisso, and S. M. Mahajan, *Phys. Plasmas* **22**, 122109 (2015).
- [38] Note that too elongated current sheets cannot form because of the occurrence of the plasmoid instability [39].
- [39] L. Comisso, M. Lingam, Y.-M. Huang, and A. Bhattacharjee, *Phys. Plasmas* **23**, 100702 (2016).
- [40] E. M. de Gouveia Dal Pino and A. Lazarian, *Astron. Astrophys.* **441**, 845 (2005).
- [41] J. Dexter, J. C. McKinney, S. Markoff, and A. Tchekhovskoy, *Mon. Not. R. Astron. Soc.* **440**, 2185 (2014).
- [42] L. H. S. Kadowaki, E. M. de Gouveia Dal Pino, and C. B. Singh, *Astrophys. J.* **802**, 113 (2015).
- [43] C. B. Singh, E. M. de Gouveia Dal Pino, and L. H. S. Kadowaki, *Astrophys. J. Lett.* **799**, L20 (2015).
- [44] B. Khiali and E. M. de Gouveia Dal Pino, *Mon. Not. R. Astron. Soc.* **455**, 838 (2015).
- [45] K. Shibata and S. Tanuma, *Earth Planets Space* **53**, 473 (2001).