Physics Letters B 765 (2017) 395–401

Contents lists available at ScienceDirect

Physics Letters B

www.elsevier.com/locate/physletb

Lovelock gravities from Born–Infeld gravity theory

P.K. Concha [a,b], N. Merino [c], E.K. Rodríguez [a,b]

[a] Departamento de Ciencias, Facultad de Artes Liberales y Facultad de Ingeniería y Ciencias, Universidad Adolfo Ibáñez, Av. Padre Hurtado 750, Viña del Mar, Chile
[b] Instituto de Ciencias Físicas y Matemáticas, Universidad Austral de Chile, Casilla 567, Valdivia, Chile
[c] Instituto de Física, Pontificia Universidad Católica de Valparaíso, Casilla 4059, Valparaíso, Chile

ARTICLE INFO

Article history:
Received 29 June 2016
Received in revised form 24 August 2016
Accepted 6 September 2016
Available online 13 September 2016
Editor: M. Cvetič

ABSTRACT

We present a Born–Infeld gravity theory based on generalizations of Maxwell symmetries denoted as $\mathfrak{C}_m$. We analyze different configuration limits allowing to recover diverse Lovelock gravity actions in six dimensions. Further, the generalization to higher even dimensions is also considered.

1. Introduction

It is a common assumption in theoretical physics that the spacetime may have more than four dimensions. This requires a generalization of General Relativity (GR) theory of gravity that includes general covariance and second order field equations for the metric. Although the Einstein–Hilbert (EH) action can be generalized to higher dimensions, the most general metric theory of gravity, satisfying the criteria of general covariance and giving second order field equations, is given by the Lanczos–Lovelock theory (LL) [1,2]. The LL action is constructed as a polynomial of degree $[D/2]$ in the Riemann curvature tensor $R^{\alpha}{}_{\beta\mu\nu}$,

$$I_{LL}[g] = \int d^D x \sum_{k=0}^{[D/2]} \alpha_k \mathcal{L}_k \,, \tag{1}$$

with

$$\mathcal{L}_k = \frac{1}{2^k} \sqrt{-g}\, \delta^{\mu_1 \dots \mu_{2k}}_{\nu_1 \dots \nu_{2k}} \, R^{\nu_1 \nu_2}_{\mu_1 \mu_2} \cdots R^{\nu_{2k-1} \nu_{2k}}_{\mu_{2k-1} \mu_{2k}} \,, \tag{2}$$

and where α_k are arbitrary constants and $\delta^{\mu_1 \dots \mu_{2k}}_{\nu_1 \dots \nu_{2k}}$ is the generalized Kronecker delta. Although the EH action is contained in the LL action, the actions with higher powers of the curvature in $D > 4$ are dynamically different from GR and are not perturbatively related.

Using first order formulation, where the affine connection $\Gamma^{\lambda}_{\mu\nu}$ is supposed to be independent from the metric $g_{\mu\nu}$, the LL theory acquires, in general, torsional degrees of freedom. This can be easily seen using Riemann–Cartan formulation of gravity in terms of the vielbein and spin connection one-forms (e^a, ω^{ab}), where $a, b = 0, 1, \dots, D$ are the local Lorentz indices. In that case the LL action can be regarded as the most general D-form invariant under local Lorentz transformations, constructed out of the vielbein e^a, the spin connection ω^{ab} and their exterior derivatives without using the Hodge dual [3,4],

$$I_{LL}[e,\omega] = \int \sum_{k=0}^{[D/2]} \alpha_k \mathcal{L}^{(k)} \,, \tag{3}$$

where α_k are arbitrary constants which are not fixed from first principles and

$$\mathcal{L}^{(k)} = \epsilon_{a_1 \cdots a_D} R^{a_1 a_2} \cdots R^{a_{2k-1} a_{2k}} e^{a_{2k+1}} \cdots e^{a_D} \,, \tag{4}$$

and where the Riemann curvature and torsion 2-forms are defined as $R^{ab} = d\omega^{ab} + \omega^a{}_c \omega^{cb}$ and $T^a = de^a + \omega^a{}_b e^b$, respectively. Then it is direct to note that a torsional dynamical field equation will arise for $k \geq 2$.

It is worth to notice that the relation between the Riemann–Cartan action (3) with the tensorial first order formalism, this latter formulated in terms of the metric and affine connection $(g_{\mu\nu}, \Gamma^{\lambda}_{\mu\nu})$, is given through $g_{\mu\nu} = \eta_{ab} e^a_\mu e^b_\nu$ and $\Gamma^{\lambda}_{\mu\nu} = \omega^{ab}_\nu e^\lambda_a e_{a\mu} + e^\lambda_a \partial_\nu e^a_\mu$. The last expression which is related with the metricity condition $\nabla^{\Gamma} g_{\mu\nu} = 0$, assures that the Riemann curvature and torsion expressed as 2-forms and tensors are essentially the same objects in both languages, i.e.,

$$R^{ab}{}_{\mu\nu} = e^a_\alpha e^b_\beta R^{\alpha\beta}{}_{\mu\nu} \,, \quad T^a_{\mu\nu} = e^a_\alpha T^\alpha_{\mu\nu} \,. \tag{5}$$

In the present work we will deal with the language of differential forms, since it makes calculation much more compact and

E-mail addresses: patillusion@gmail.com (P.K. Concha), nemerino@gmail.com (N. Merino), everodriguezd@gmail.com (E.K. Rodríguez).

http://dx.doi.org/10.1016/j.physletb.2016.09.008

because it is more suitable to describe the gauge structure that the theory possesses.

As shown in Ref. [5], requiring the theory to have the maximum possible number of degrees of freedom fixes, in the first order formalism, the α_k constants in (3). In odd dimensions, the Lagrangian becomes a Chern–Simons (CS) form [6,7], which is a functional $\mathcal{L}_{CS}[A]$ of a gauge connection one-form A containing the vielbein and spin connection. The corresponding CS action is invariant, up to a boundary term, under a bigger symmetry (dS, AdS or Poincaré groups). In even dimensions the same requirement leads to a Born–Infeld (BI) action which is also constructed in terms of the curvature associated with the gauge connection, but it is locally invariant only under the Lorentz subgroup.

Another interesting family, called Pure Lovelock (PL) gravity, has been recently proposed in [8–10] as another way of fixing the α's. It consists of only two terms, the cosmological one and a single p-power in the curvature, with $p = 1, \ldots, N = [(D-1)/2]$. Remarkably, their black holes solutions behave asymptotically like the ones in GR and like the dimensionally continued black holes [11,12] near the horizon.

In Ref. [4] it was suggested that the metric LL theory should have the same degrees of freedom (dof) as the higher-dimensional EH gravity, i.e., $D(D-3)/2$. However, the non-linearity of the theory makes the symplectic matrix to change the rank with the backgrounds [13] generating extra local symmetries and decreasing degrees of freedom in some of them. This behavior, typical for LL theories, has also been found in Lovelock–Chern–Simons gravities [14,15] and recently in PL gravity [16] where the number of dof changes with the backgrounds between 0 and $D(D-3)/2$. Besides, this property is not only intrinsic of the Riemann sector, but also happens when torsional degrees of freedom are considered. This is the case when one looks for charged black hole solutions in CS supergravity [17].

On the other hand, the supersymmetric version of the LL theory is not known in general, except for few cases such as the EH and CS ones. The existence of a supersymmetric version in $D = 5$ for non-vanishing constants α_1 and α_2 is discussed in Ref. [18], even though its explicit form is still unknown. It has also been suggested in [19] that a supersymmetric version of PL theory might be constructed using new symmetries obtained through expansion methods of Lie algebras [20–22]. Indeed, those methods have already been used to relate diverse gravity theories. For example, it has been found that even and odd-dimensional GR can be obtained as a special limit of BI and CS theories, constructed with expansions of the $\mathfrak{so}(D-1,2)$ algebra [23–27].

Recently, in Ref. [19], it has been shown that the PL action in odd dimensions can be obtained as a limit from a CS action based on a special expansion of the $\mathfrak{so}(D-1,2)$ algebra, denoted by $\mathfrak{C}_m$. Those symmetries were introduced in Refs. [28–34] and can be regarded as generalizations of the so called Maxwell algebra [35,36], which describes the symmetries of quantum fields in Minkowski space with the presence of a constant electromagnetic field. Thus, for completeness and also due to the growing interest in the effect of higher-curvature terms in the holographic context (see for example [37–40]), in this work we will show that different Lovelock gravity actions in even dimensions can be obtained from a BI action based on the $\mathfrak{C}_m$ algebra. We shall start by considering the six-dimensional spacetime since it allows us to obtain a bigger variety of gravity theories. Indeed, by applying in four dimensions the prescription presented here would lead only to the Einstein gravity with cosmological constant term.

The present work is organized as follows: in Section 2 we briefly review the BI gravity theory. Section 3 and 4 contain our main results. We present the explicit expression for the six-dimensional BI type gravity action based on the $\mathfrak{C}_7$ two-form curvature. The general setup in order to derive different Lovelock gravity action in a particular limit is given. We conclude our work by providing the generalization to higher even dimensions.

2. Brief review about Born–Infeld gravity theory

As was previously pointed out the Lanczos–Lovelock theory refers to a family parametrized by a set of real coefficients α_k, which are not fixed from first principles. To require the theory possess the largest possible number of degrees of freedom, fixes the α_k parameters in terms of the gravitational and the cosmological constants [5]. As a result, in even dimensions the action has a Born–Infeld form invariant only under local Lorentz rotations, in the same way as the EH action. In this section, we review the main aspects of the BI gravity theory. As was shown in Ref. [5], choosing the coefficients as

$$\alpha_k = \alpha_0 (2\gamma)^k \binom{n}{k}, \qquad (6)$$

with $0 \leq k \leq n$ and

$$\alpha_0 = \frac{\kappa}{(2n)\, l^{2n}}, \quad \gamma = -sign(\Lambda)\frac{l^2}{2},$$

the LL Lagrangian leads to the so-called Lovelock–Born–Infeld (LBI) Lagrangian [41,42] in $D = 2n$

$$\mathcal{L}_{BI}^{2n} = \frac{\kappa}{2n}\epsilon_{a_1\cdots a_{2n}}\bar{R}^{a_1a_2}\cdots\bar{R}^{a_{2n-1}a_{2n}}, \qquad (7)$$

where $\bar{R}^{ab} = R^{ab} + \frac{1}{l^2}e^a e^b$ corresponds to the AdS curvature. Here R^{ab} is the usual Lorentz curvature and l is a length parameter. Let us notice that the Lagrangian (7) is the Pfaffian of the 2-form $\bar{R}^{ab}$ and can be rewritten as,

$$\mathcal{L}_{BI}^{2n} = 2^{n-1}(n-1)!\kappa\sqrt{\det\left(R^{ab} + \frac{1}{l^2}e^a e^b\right)}, \qquad (8)$$

which reminds us the Born–Infeld electrodynamics Lagrangian.

The BI gravity Lagrangian, which is basically constructed under the requirement of having a unique maximally degenerate AdS vacuum, has the advantage of having well defined black holes configurations. The family of gravity actions constructed with the coefficients (6) up to a certain fixed k, with $0 \leq k \leq n$, are characterized by the fact that they have a unique k-order degenerate AdS vacuum, with the BI Lagrangian case being described by $k = n$. As shown in [11,12], such family (which includes EH and EGB gravities for $k = 1, 2$) is free of degeneracies in the static sherically symmetric sector of the space of solutions, i.e., they have well defined black holes configurations. This does not occur for the Lovelock theory for which, for arbitrary α_k constants, the field equations do not determine completely the components of the curvature and torsion in the static sector. Besides, the BI gravity Lagrangian possesses a large number of appealing features like cosmological models, black hole solutions, etc.

It is important to note that the Lagrangian (7) is invariant only under local Lorentz transformations and not under the AdS group. In this way, in $D = 2n$ the Levi-Civita symbol $\epsilon_{a_1\cdots a_{2n}}$ in (7) can be regarded as the only invariant tensor under the Lorentz group $SO(2n-1,1)$. This choice of the invariant tensor, which is necessary in order to reproduce a non-trivial action principle, breaks the full AdS symmetry to its Lorentz subgroup. In fact, the BI gravity action can be written as follows

$$I_{BI}^{2n} = \kappa\int\langle F\wedge\cdots\wedge F\rangle = \kappa\int F^{A_1}\wedge\cdots\wedge F^{A_n}\left\langle T_{A_1}\cdots T_{A_n}\right\rangle. \qquad (9)$$

Here F is the AdS 2-form curvature

$$F = F^A T_A = \frac{1}{2}\left(R^{ab} + \frac{1}{l^2}e^a e^b\right) J_{ab} + \frac{1}{l} T^a P_a ,$$

and $\langle T_{A_1} \cdots T_{A_n} \rangle$ is chosen as an invariant tensor for the Lorentz group only,

$$\langle T_{A_1} \cdots T_{A_n} \rangle = \langle J_{a_1 a_2} \cdots J_{a_{2n-1} a_{2n}} \rangle = \frac{2^{n-1}}{n} \epsilon_{a_1 \cdots a_{2n}} . \tag{10}$$

Then the action (9) can be expressed as

$$I_{BI}^{2n} = \int \sum_{k=0}^{n} \frac{\kappa}{2n} \binom{n}{k} l^{2k-2n} \epsilon_{a_1 \cdots a_{2n}} R^{a_1 a_2} \cdots R^{a_{2k-1} a_{2k}} e^{a_{2k+1}} \cdots e^{a_{2n}} . \tag{11}$$

In $D = 4$ the BI action is written as a particular linear combination of the standard Einstein–Hilbert action with cosmological constant and the Euler density,

$$I_{BI}^{4D} = \frac{\kappa}{4} \int \epsilon_{abcd} \left(R^{ab} R^{cd} + \frac{2}{l^2} R^{ab} e^c e^d + \frac{1}{l^4} e^a e^b e^c e^d \right) . \tag{12}$$

In the same way, the Lovelock coefficients α_0, α_1, α_2 and α_3 are chosen so that the $D = 6$ BI gravity action is given by

$$I_{BI}^{6D} = \frac{\kappa}{6} \int \epsilon_{abcdef} \left(R^{ab} R^{cd} R^{ef} + \frac{3}{l^2} R^{ab} R^{cd} e^e e^f + \frac{3}{l^4} R^{ab} e^c e^d e^e e^f + \frac{1}{l^6} e^a e^b e^c e^d e^e e^f \right) . \tag{13}$$

3. $D = 6$ Lovelock gravity actions from Born–Infeld type theory

In this section, we show the explicit construction of a BI type theory based on enlarged symmetries and its relation to different six-dimensional Lovelock gravity actions.

3.1. Why $\mathfrak{C}_m$ algebras?

Our objective requires to find a symmetry which allows to separate each term of the original Lovelock Lagrangian in different sectors. Thus, under a specific limit, the unwanted sector can be avoided leading to interesting gravity actions. To this purpose, every Lovelock term should be originated by different components of an invariant tensor leading to a Born–Infeld type action. These desired properties have origin in the $\mathfrak{so}(D-1,2) \oplus \mathfrak{so}(D-1,1)$ Lie algebra[1] [28–31] which has been generalized, using the abelian semigroup expansion method (S-expansion) [21], to a family of Maxwell type algebras denoted as $\mathfrak{C}_m$ [32,33]. Their supersymmetric extensions have also been constructed in Refs. [43,44].

As was shown in Ref. [19] the $\mathfrak{C}_m$ algebras are obtained from AdS considering $S_M^{(m-2)} = \{\lambda_0, \lambda_1, \ldots, \lambda_{m-2}\}$ as the relevant semigroup, whose multiplication law is given by

$$\lambda_\alpha \lambda_\beta = \begin{cases} \lambda_{\alpha+\beta}, & \text{if } \alpha + \beta \le m-2 , \\ \lambda_{\alpha+\beta-2\left[\frac{m-1}{2}\right]}, & \text{if } \alpha+\beta > m-2 . \end{cases} \tag{14}$$

After extracting a resonant subalgebra, one finds the $\mathfrak{C}_m$ algebra whose generators satisfy the following commutation relations

$$\begin{aligned}
\left[J_{ab,(i)}, J_{cd,(j)}\right] &= \eta_{bc} J_{ad,(i+j) \bmod \left[\frac{m-1}{2}\right]} - \eta_{ac} J_{bd,(i+j) \bmod \left[\frac{m-1}{2}\right]} \\
&\quad - \eta_{bd} J_{ac,(i+j) \bmod \left[\frac{m-1}{2}\right]} + \eta_{ad} J_{bc,(i+j) \bmod \left[\frac{m-1}{2}\right]} , \\
\left[J_{ab,(i)}, P_{a,(k)}\right] &= \eta_{bc} P_{a,(i+k) \bmod \left[\frac{m-1}{2}\right]} - \eta_{ac} P_{b,(i+k) \bmod \left[\frac{m-1}{2}\right]} , \\
\left[P_{a,(i)}, P_{b,(k)}\right] &= J_{ab,(i+k+1) \bmod \left[\frac{m-1}{2}\right]} .
\end{aligned} \tag{15}$$

The new generators $\{J_{ab,(i)}, P_{a,(k)}\}$ are related to the $\mathfrak{so}(5,2)$ ones $\{\tilde{J}_{ab}, \tilde{P}_a\}$ through

$$\begin{aligned}
J_{ab,(i)} &= \lambda_{2i} \otimes \tilde{J}_{ab} , \\
P_{a,(k)} &= \lambda_{2k+1} \otimes \tilde{P}_a ,
\end{aligned}$$

with $i = 0, 1, \ldots, \left[\frac{m-2}{2}\right]$ and $k = 0, 1, \ldots, \left[\frac{m-3}{2}\right]$.

The two-form curvature $F = dA + A \wedge A$ for the $\mathfrak{C}_m$ algebra is given by

$$F = F^A T_A = \frac{1}{2} \sum_i \mathcal{R}^{ab,(i)} J_{ab,(i)} + \frac{1}{\ell} \sum_k R^{a,(k)} P_{a,(k)} , \tag{16}$$

where

$$\begin{aligned}
\mathcal{R}^{ab,(i)} &= d\omega^{ab,(i)} + \delta^i_{(j+l) \bmod \left[\frac{m-1}{2}\right]} \omega^{a,(j)}_{\ c} \omega^{cb,(l)} \\
&\quad + \frac{1}{\ell^2} \delta^i_{(p+q+1) \bmod \left[\frac{m-1}{2}\right]} e^{a,(p)} e^{b,(q)} , \\
R^{a,(k)} &= de^{a,(k)} + \delta^k_{(j+p) \bmod \left[\frac{m-1}{2}\right]} \omega^{a,(j)}_{\ c} e^{c,(p)} .
\end{aligned}$$

Let us note that $\omega^{ab,(0)}$ and $e^{a,(0)}$ correspond to the spin connection ω^{ab} and the vielbein e^a, respectively.

In order to build a six-dimensional BI type gravity action based on the $\mathfrak{C}_m$ two-form curvature, we require the explicit expression of the invariant tensor. Fortunately, the S-expansion method offers the possibility of deriving the invariant tensor for the expanded algebra from the original one. Indeed, following the Theorem VII.2 of Ref. [21], the non-vanishing components of an invariant tensor are given by

$$\langle J_{ab,(i)} J_{cd,(l)} J_{ef,(m)} \rangle = \frac{4}{3} \sigma_{2j} \delta^j_{(i+l+m) \bmod \left[\frac{m-1}{2}\right]} \epsilon_{abcdef} , \tag{17}$$

where σ_{2j} are arbitrary constants. As in the original BI case, this choice of the invariant tensor breaks the $\mathfrak{C}_m$ group to a Lorentz type subgroup generated by $\{J_{ab,(i)}\}$.

Since the original Lovelock terms will arise in the BI type action through $\mathcal{R}^{ab,(0)}$ and $\mathcal{R}^{ab,(1)}$, it is straightforward to know from (17) in which sector every Lovelock term will appear. The following table clarifies this point:

	σ_0	σ_2	σ_4	σ_6
$\mathfrak{C}_4$	RRR	$RRee$, $Reeee$, $eeeeee$		
$\mathfrak{C}_5$	RRR, $Reeee$	$RRee$, $eeeeee$		
$\mathfrak{C}_6$	RRR	$RRee$, $eeeeee$	$Reeee$	
$\mathfrak{C}_7$	RRR, $eeeeee$	$RRee$	$Reeee$	
$\mathfrak{C}_{m\geq 8}$	RRR	$RRee$	$Reeee$	$eeeeee$

As $RRR = \epsilon_{abcdef} R^{ab} R^{cd} R^{ef}$ corresponds to a boundary term, the minimal algebra which allows to separate each Lovelock term in different sectors of the BI type action corresponds to the $\mathfrak{C}_7$ algebra.

[1] Also known as AdS–Lorentz algebra.

3.2. $D = 6$ Lovelock gravity actions and $\mathfrak{C}_7$ algebra

Considering the $\mathfrak{C}_7$ algebra, we present different limits and conditions on the σ's leading to various Lovelock gravity actions in $D = 6$.

Let us first consider the $\mathfrak{C}_7$-valued connection one-form

$$A = \frac{1}{2}\omega^{ab} J_{ab} + \frac{1}{\ell}e^a P_a + \frac{1}{2}k^{ab} Z_{ab} + \frac{1}{\ell}h^a Z_a + \frac{1}{2}\tilde{k}^{ab} \tilde{Z}_{ab} + \frac{1}{\ell}\tilde{h}^a \tilde{Z}_a , \tag{18}$$

and the associated curvature two-form $F = dA + A \wedge A$

$$F = F^A T_A = \frac{1}{2}\mathcal{R}^{ab} J_{ab} + \frac{1}{\ell}R^a P_a + \frac{1}{2}F^{ab} Z_{ab} + \frac{1}{\ell}H^a Z_a + \frac{1}{2}\tilde{F}^{ab} \tilde{Z}_{ab} + \frac{1}{\ell}\tilde{H}^a \tilde{Z}_a , \tag{19}$$

where

$$\mathcal{R}^{ab} = R^{ab} + 2k^{[a|}_{\ c}\tilde{k}^{c|b]} + \frac{1}{\ell^2}\left(h^a h^b + 2e^a \tilde{h}^b\right),$$
$$R^a = T^a + k^a_{\ c}\tilde{h}^c + \tilde{k}^a_{\ c}h^c ,$$
$$F^{ab} = Dk^{ab} + \tilde{k}^a_{\ c}\tilde{k}^{cb} + \frac{1}{\ell^2}\left(e^a e^b + 2h^a \tilde{h}^b\right),$$
$$H^a = Dh^a + k^a_{\ c}e^c + \tilde{k}^a_{\ c}\tilde{h}^c ,$$
$$\tilde{F}^{ab} = D\tilde{k}^{ab} + 2k^a_{\ c}k^{cb} + \frac{1}{\ell^2}\left(\tilde{h}^a \tilde{h}^b + 2e^a h^b\right),$$
$$\tilde{H}^a = D\tilde{h}^a + k^a_{\ c}h^c + \tilde{k}^a_{\ c}e^c .$$

Here $D = d + \omega$ denotes the Lorentz covariant exterior derivative and R^{ab} is the usual Lorentz curvature.

Following eq. (17), it is possible to show that the only non-vanishing components of an invariant tensor required to build a BI gravity action, are given by

$$\begin{aligned}
&\langle J_{ab} J_{cd} J_{ef}\rangle = \frac{4}{3}\sigma_0\epsilon_{abcdef} && \langle Z_{ab} Z_{cd} Z_{ef}\rangle = \frac{4}{3}\sigma_0\epsilon_{abcdef}\\
&\langle J_{ab} Z_{cd} \tilde{Z}_{ef}\rangle = \frac{4}{3}\sigma_0\epsilon_{abcdef} && \langle \tilde{Z}_{ab} \tilde{Z}_{cd} \tilde{Z}_{ef}\rangle = \frac{4}{3}\sigma_0\epsilon_{abcdef}\\
&\langle J_{ab} J_{cd} Z_{ef}\rangle = \frac{4}{3}\sigma_2\epsilon_{abcdef} && \langle J_{ab} \tilde{Z}_{cd} \tilde{Z}_{ef}\rangle = \frac{4}{3}\sigma_2\epsilon_{abcdef}\\
&\langle Z_{ab} Z_{cd} \tilde{Z}_{ef}\rangle = \frac{4}{3}\sigma_2\epsilon_{abcdef} && \langle Z_{ab} \tilde{Z}_{cd} \tilde{Z}_{ef}\rangle = \frac{4}{3}\sigma_4\epsilon_{abcdef}\\
&\langle J_{ab} J_{cd} \tilde{Z}_{ef}\rangle = \frac{4}{3}\sigma_4\epsilon_{abcdef} && \langle J_{ab} Z_{cd} Z_{ef}\rangle = \frac{4}{3}\sigma_4\epsilon_{abcdef}
\end{aligned} \tag{20}$$

where σ_0, σ_2 and σ_4 are dimensionless arbitrary constants. As was previously mentioned, this choice of invariant tensor breaks the $\mathfrak{C}_7$ algebra to its Lorentz type subalgebra $\mathfrak{L}^{\mathfrak{C}_7}$ generated by $\left\{J_{ab}, Z_{ab}, \tilde{Z}_{ab}\right\}$.

Then, considering the curvature two-form (19) and the non-vanishing components of the invariant tensor (20) in the general expression of a six-dimensional Born–Infeld type gravity action

$$I^{6D}_{\mathfrak{L}^{\mathfrak{C}_7}-BI} = \kappa \int \langle FFF\rangle = \kappa \int F^A F^B F^C \langle T_A T_B T_C\rangle ,$$

we find

$$\begin{aligned}
I^{6D}_{\mathfrak{L}^{\mathfrak{C}_7}-BI} = \frac{\kappa}{6}\epsilon_{abcdef}\int \sigma_0\Big[&\mathcal{R}^{ab}\mathcal{R}^{cd}\mathcal{R}^{ef} + 6\mathcal{R}^{ab}F^{cd}\tilde{F}^{ef}\\
&+ F^{ab}F^{cd}F^{ef} + \tilde{F}^{ab}\tilde{F}^{cd}\tilde{F}^{ef}\Big]\\
&+ \sigma_2\left[3\mathcal{R}^{ab}\mathcal{R}^{cd}F^{ef} + 3\mathcal{R}^{ab}\tilde{F}^{cd}\tilde{F}^{ef} + 3F^{ab}F^{cd}\tilde{F}^{ef}\right]\\
&+ \sigma_4\left[3\mathcal{R}^{ab}\mathcal{R}^{cd}\tilde{F}^{ef} + 3\mathcal{R}^{ab}F^{cd}F^{ef} + 3F^{ab}\tilde{F}^{cd}\tilde{F}^{ef}\right] .
\end{aligned} \tag{21}$$

Separating the purely gravitational terms (ω, e) from those containing extra fields $\left(k, \tilde{k}, h, \tilde{h}\right)$ the action (21) can be rewritten as

$$\begin{aligned}
I^{6D}_{\mathfrak{L}^{\mathfrak{C}_7}-BI} = \frac{\kappa}{6}\int \sigma_0\Big[&\epsilon_{abcdef}R^{ab}R^{cd}R^{ef} + \frac{1}{\ell^6}\epsilon_{abcdef}e^a e^b e^c e^d e^e e^f\\
&+ \tilde{\mathcal{L}}_0\left(\omega, e, k, \tilde{k}, h, \tilde{h}\right)\Big]\\
&+ \sigma_2\left[\frac{3}{\ell^2}\epsilon_{abcdef}R^{ab}R^{cd}e^e e^f + \tilde{\mathcal{L}}_2\left(\omega, e, k, \tilde{k}, h, \tilde{h}\right)\right]\\
&+ \sigma_4\left[\frac{3}{\ell^4}\epsilon_{abcdef}R^{ab}e^c e^d e^e e^f + \tilde{\mathcal{L}}_4\left(\omega, e, k, \tilde{k}, h, \tilde{h}\right)\right] .
\end{aligned} \tag{22}$$

Omitting the boundary term, each term of the Lovelock series appears in a different sector of the BI type gravity action. The same feature occurs in the seven-dimensional Chern–Simons case using the $\mathfrak{C}_7$ algebra [19]. Nevertheless, unlike the odd-dimensional case, the action is not gauge invariant of the entire gauge group but only under a Lorentz type subgroup.

Let us note that the action (22) contains vielbein type fields $\left(h, \tilde{h}\right)$ leading to products of different vielbeins. This action has an analogous structure to dRGT (de Rham, Gabadadze, Tolley) massive gravity theories in the vielbein formalism [45,46] suggesting a tri-metric theory in six dimensions, each of them with an EH type term. A smaller algebra ($\mathfrak{C}_5$) would lead only to two kind of vielbein (e, h) leading to Lagrangian terms analogous to the bi-gravity formalism [47,48]. It would be interesting to investigate under what conditions they might be connected to our results. However, important differences should be pointed out. Indeed such theories are in the second order formalism, whilst our description is a first order description. Additionally, we have introduced h's only as extra fields and it is not the original purpose to interpret them as new vielbeins to describe a n-gravity theory. Besides, since our purpose is to analyze the limits where those extra fields vanish, we postpone that discussion for future work.

Interestingly diverse gravity actions can be obtained following appropriate conditions on the σ's and applying suitable limits on the extra fields.

3.2.1. Pure Lovelock gravity action

The $p = 1$ PL action, which trivially corresponds to $EH + \Lambda$, emerges from the BI type gravity action (22) imposing $\sigma_2 = 0$, $\sigma_0 = \sigma_4$ and considering a matter-free configuration $\left(k = \tilde{k} = h = \tilde{h} = 0\right)$

$$I^{6D}_{BI\to PL} = \kappa\sigma_0 \int \frac{1}{2\ell^4}\epsilon_{abcdef}R^{ab}e^c e^d e^e e^f + \frac{1}{6\ell^6}\epsilon_{abcdef}e^a e^b e^c e^d e^e e^f , \tag{23}$$

where we have omitted the boundary term $\epsilon_{abcdef}R^{ab}R^{cd}R^{ef}$. This feature is well desired since if BI type gravity theories are the appropriate theories to describe gravity then such theories should satisfy the correspondence principle. This property will be our principal requirement in a possible supersymmetric extension of PL theory.

Not only the lowest order PL action can be recovered from the BI type action but also the maximal one ($p = 2$). Indeed, when the σ_4 constant vanishes and $\sigma_0 = -\sigma_2$, the matter-free configuration limit $\left(k = \tilde{k} = h = \tilde{h} = 0\right)$ leads to

$$I^{6D}_{BI\to \max PL} = \kappa\sigma_0 \int \frac{1}{2\ell^2}\epsilon_{abcdef} R^{ab} R^{cd} e^e e^f - \frac{1}{6\ell^6}\epsilon_{abcdef} e^a e^b e^c e^d e^e e^f \ . \tag{24}$$

As in the CS case, although we obtain the PL action, the dynamical limit requires a more subtle treatment. A detailed discussion about the dynamical issue in odd dimensions has been done in Ref. [19].

3.2.2. $EH + LL$ gravity action

Another non-trivial election of the σ's leads to an alternative Lovelock gravity action. The $EH + LL$ action, where LL is an arbitrary Lanczos–Lovelock term [18], can be derived from the BI type action (22) imposing $\sigma_0 = 0$, $\sigma_2 = \sigma_4$, and considering a matter-free configuration limit $\left(k = \tilde{k} = h = \tilde{h} = 0\right)$:

$$I^{6D}_{BI\to EH+LL} = \kappa\sigma_2 \int \frac{1}{2\ell^4}\epsilon_{abcdef} R^{ab} e^c e^d e^e e^f + \frac{1}{2\ell^2}\epsilon_{abcdef} R^{ab} R^{cd} e^e e^f \ . \tag{25}$$

Thus, the action contains the EH term and the Gauss–Bonnet (GB) term. Naturally, in six dimensions this is the only possibility to relate the Einstein–Hilbert term with another higher power in the curvature. Nevertheless, as we will see in the $2n$-dimensional case, the presence of higher-curvature terms in the BI type action will allow to equip the EH term with an arbitrary p-order LL term ($p \neq 0$).

It is important to point out that the supersymmetric extension of the $EH + LL$ gravity remains unsolved. Although some discussions can be found in Ref. [18], the explicit form of the supergravity action is still a mystery. The procedure presented here could be generalized to supergravity allowing to find a wider class of supergravity theories.

On the other hand, as it was mentioned in Ref. [49], the causality is violated for quadratic gravity theories when the GB coupling is finite. One could argue that considering other spin-2 fields could fix the causality violation, however this would lead to restrictive field equations where not even pp-wave could satisfy. Indeed, the situation is quite different to the one presented in Ref. [50] where no copy of the EH terms appears. In our present case, the $\mathfrak{C}_m$ symmetries imply to copy every original Lovelock term avoiding the possibility to find Gauss–Bonnet equations when h or $\tilde{h}$ is identified as the true vielbein. This does not occur in the PL case where a non-trivial identification of the extra fields allows to reproduce the appropriate dynamics [19].

3.2.3. Lovelock gravity with generalized cosmological constant

A six-dimensional gravity action in presence of a generalized cosmological constant term [32,51] can be recovered in a particular limit. Indeed, when $\tilde{k}^{ab} = \tilde{h}^a = h^a = 0$, the BI type action reduces to

$$\begin{aligned} I^{6D}_{BI\to EH+LL+\tilde{\Lambda}} &= \kappa\sigma_0 \int \epsilon_{abcdef}\left[\frac{1}{6l^6} e^a e^b e^c e^d e^e e^f + \frac{1}{2l^2} Dk^{ab} Dk^{cd} e^e e^f + \frac{1}{2l^4} Dk^{ab} e^c e^d e^e e^f\right] \\ &\quad + \epsilon_{abcdef}\left[\frac{1}{2l^2} R^{ab} R^{cd} e^e e^f\right] \\ &\quad + \epsilon_{abcdef}\left[\frac{1}{2l^4} R^{ab} e^c e^d e^e e^f + \frac{1}{l^2} R^{ab} Dk^{cd} e^e e^f\right] \\ &= \kappa\sigma_0 \int L_{EH} + L_{LL} + L_{\tilde{\Lambda}} \ , \end{aligned} \tag{26}$$

where we have omitted the boundary terms and where we have set $\sigma_0 = \sigma_2 = \sigma_4$.

The $L_{\tilde{\Lambda}}$ term includes the cosmological constant term plus additional pieces depending on the extra field k^{ab}. This can be seen as a generalization of the result obtained for the Maxwell symmetry [51] to six dimensions. Interestingly, in this limit, the action corresponds to the six-dimensional BI type constructed out of the curvature two-form for the $\mathfrak{so}(D-1,2) \oplus \mathfrak{so}(D-1,1)$ algebra.

Let us observe that the same result can be obtained imposing $k^{ab} = 0$ instead of $\tilde{k}^{ab} = 0$. Obviously, the additional terms appearing in the action would depend in such case on $\tilde{k}^{ab}$. The presence of the extra fields k^{ab} or $\tilde{k}^{ab}$ leads to an alternative extension of standard gravity allowing the introduction of a generalized cosmological term. At the supersymmetric level, an analogous result has been presented in Ref. [44] using a supersymmetric extension of the $\mathfrak{so}(D-1,2) \oplus \mathfrak{so}(D-1,1)$ algebra.

4. Extension to $D = 2n$ gravities

In this section we present the $2n$-dimensional BI type gravity action based on the $\mathfrak{C}_{2n+1}$ algebra. Additionally, we provide with the suitable limits on the extra fields and the general conditions on the σ's necessary to recover diverse gravity actions.

Following the same discussion of the previous section, one can see that the minimal symmetry allowing to separate each Lovelock term in different sectors corresponds to $\mathfrak{C}_{2n+1}$. From Theorem VII.2 of Ref. [21], the non-vanishing components of an invariant tensor necessary to build a $2n$-dimensional BI type gravity action based on the $\mathfrak{C}_{2n+1}$ curvature two-form are given by

$$\left\langle J_{a_1a_2,(i_1)} \cdots J_{a_{2n-1}a_{2n},(i_n)}\right\rangle = \frac{2^{n-1}}{n}\alpha_{2j}\delta^{j}_{(i_1+\cdots+i_n)\bmod[n]}\epsilon_{a_1\cdots a_{2n}} \ , \tag{27}$$

with $i = 0, 1, \ldots, \left[\frac{2n-1}{2}\right]$. This invariant tensor breaks the $\mathfrak{C}_{2n+1}$ symmetry to a Lorentz type subgroup $\mathfrak{L}^{\mathfrak{C}_{2n+1}}$ generated by $\left\{J_{ab,(i)}\right\}$. It is important to clarify that an invariant tensor of the whole $\mathfrak{C}_{2n+1}$ group would lead to a topological invariant.

Considering the general expression for the curvature two-form (16) and the non-vanishing component of the invariant tensor (27) in the general expression of a $2n$-dimensional Born–Infeld type gravity action (see eq. (9)) we find

$$\begin{aligned} I^{2n}_{\mathfrak{L}^{\mathfrak{C}_{2n+1}}-BI} &= \int \sum_{k=1}^{n} l^{2k-2}\frac{\kappa}{2n}\binom{n}{k}\sigma_{2j}\delta^{j}_{(i_1+\cdots+i_n)\bmod[n]}\delta^{i_{k+1}}_{p_1+q_1}\cdots\delta^{i_n}_{p_{n-k}+q_{n-k}} \\ &\quad \times \varepsilon_{a_1\cdots a_{2n}} R^{(a_1a_2,i_1)}\ldots R^{(a_{2k-1}a_{2k},i_k)} \\ &\quad \times e^{(a_{2k+1},p_1)} e^{(a_{2k+2},q_1)}\ldots e^{(a_{2n-1},p_{n-k})} e^{(a_{2n},q_{n-k})} \ . \end{aligned} \tag{28}$$

Separating the gravitational terms (ω, e) from those containing extra fields $\left(\omega^{ab,(i)}, e^{a,(i)}\ \text{for}\ i \neq 0\right)$, the action (28) can be rewritten as

$$\begin{aligned} I^{2n}_{\mathfrak{L}^{\mathfrak{C}_{2n+1}}-BI} = \frac{\kappa}{2n}\int \sigma_0\Big[&\epsilon_{a_1a_2\cdots a_{2n}} R^{a_1a_2}\ldots R^{a_{2n-1}a_{2n}} \\ &+ \frac{1}{\ell^{2n}}\epsilon_{a_1a_2\cdots a_{2n}} e^{a_1} e^{a_2}\ldots e^{a_{2n}} + \tilde{\mathcal{L}}_0\left(\omega^{ab,(i)}, e^{a,(i)}\right)\Big] \\ &+ \sigma_2\Big[\frac{n}{\ell^2}\epsilon_{a_1a_2\cdots a_{2n}} R^{a_1a_2}\ldots R^{a_{2n-3}a_{2n-2}} e^{a_{2n-1}} e^{a_{2n}} \\ &+ \tilde{\mathcal{L}}_2\left(\omega^{ab,(i)}, e^{a,(i)}\right)\Big] \end{aligned}$$

$$
\begin{aligned}
&+\cdots \\
&+\sigma_{2n-2}\left[\frac{n}{\ell^{2n-2}}\epsilon_{a_1a_2\cdots a_{2n}}R^{a_1a_2}e^{a_3}\ldots e^{a_{2n}}\right. \\
&\left.+\tilde{\mathcal{L}}_{2n-2}\left(\omega^{ab,(i)},e^{a,(i)}\right)\right]. \qquad (29)
\end{aligned}
$$

As in the six-dimensional case, there are diverse configurations allowing to recover different gravity theories. In the following table we present the explicit conditions on the σ's and the pertinent limit on the extra fields required to derive diverse gravity actions in $2n$ dimensions:

Gravity actions	σ's conditions	Matter-free configuration limit
p-order Pure Lovelock	$\sigma_0=(\pm 1)^{p+1}\sigma_{2n-2p}\neq 0, \sigma_i=0$	$\omega^{ab,(i)}=e^{a,(i)}=0$ for $i\neq 0$
$EH+p$-order LL	$\sigma_{2n-2}=\sigma_{2n-2(p+1)}\neq 0, \sigma_i=0$	$\omega^{ab,(i)}=e^{a,(i)}=0$ for $i\neq 0$
$EH+LL+\tilde{\Lambda}$	$\sigma_i=\sigma_{i+1}\neq 0,$ for $i=0,\ldots,\left[\frac{2n-3}{2}\right]$	$\omega^{ab,(i+1)}=e^{a,(i)}=0$ for $i\neq 0$

In particular, $\tilde{\Lambda}$ refers to a generalized cosmological term which, besides containing the standard cosmological constant term, includes additional pieces depending on the extra field $\omega^{ab,(1)}$. This generalizes to $2n$ dimensions the results obtained in Refs. [32,51]. Let us note that, in the six-dimensional case, the extra field has been identified as k^{ab}.

5. Summary and discussions

In the present work, we have shown that deforming the BI gravity theory (based on the AdS curvature) using the abelian semigroup expansion allows to reproduce diverse Lovelock gravity theories considering appropriate matter-free configuration limit and imposing pertinent conditions on the σ constants.

The relations among the expanded BI gravity theories and the Lovelock one are motivated by the recent connection between even-dimensional standard GR and expanded BI gravity theories [24,25] where the properties of the S-expansion were used. Diverse results have been obtained, not only in the BI context, using the Lie Algebra expansion method. In fact, it was shown in Ref. [50], that EH Lagrangian can be derived from an expanded AdS CS gravity theory. Additionally the proper Einstein dynamics can also be obtained in an appropriate coupling constant limit using the S-expansion procedure [23].

The results obtained along this paper not only present some new explicit relations among BI and Lovelock theories but also could bring valuable information in order to derive other (super)gravities. Indeed, various supersymmetric extensions of (pure)Lovelock gravity theories have not been explored yet and their standard constructions remain to be a highly difficult task. However, our results suggest that the super (pure)Lovelock could be related to supergravity theory with expanded Lie algebras. Besides, the same procedure could be applied to CS supergravity theories.

Acknowledgments

Research of PKC and EKR is supported by the Newton–Picarte CONICYT Grant No. DPI20140053. NM was supported by the Chilean FONDECYT Project 3130445. He is also grateful with the Becas-Chile postdoctoral grant, whose financial support helped in the last stage of this work. PKC and EKR wish to thank A. Anabalón for his kind hospitality at Departamento de Ciencias of Universidad Adolfo Ibañez. The authors are also grateful to R. Durka for useful comments.

References

[1] D. Lovelock, The Einstein tensor and its generalizations, J. Math. Phys. 12 (1971) 498.
[2] C. Lanczos, The four-dimensionality of space and the Einstein tensor, J. Math. Phys. 13 (1972) 874.
[3] B. Zumino, Gravity theories in more than four dimensions, Phys. Rep. 137 (1986) 109.
[4] C. Teitelboim, J. Zanelli, Dimensionally continued topological gravitation theory in Hamiltonian form, Class. Quantum Gravity 4 L125 (1987).
[5] R. Troncoso, J. Zanelli, Higher dimensional gravity, propagating torsion and AdS gauge invariance, Class. Quantum Gravity 17 (2000) 4451, arXiv:hep-th/9907109.
[6] A.H. Chamseddine, Topological gauge theory of gravity in five-dimensions and all odd dimensions, Phys. Lett. B 223 (1989) 291.
[7] A.H. Chamseddine, Topological gravity and supergravity in various dimensions, Nucl. Phys. B 346 (1990) 213.
[8] R.G. Cai, N. Ohta, Black holes in pure Lovelock gravities, Phys. Rev. D 74 (2006) 064001, arXiv:hep-th/0604088.
[9] N. Dadhich, J.M. Pons, K. Prabhu, On the static Lovelock black holes, Gen. Relativ. Gravit. 45 (2013) 1131, arXiv:1201.4994 [gr-qc].
[10] N. Dadhich, J.M. Pons, K. Prabhu, Thermodynamical universality of the Lovelock black holes, Gen. Relativ. Gravit. 44 (2012) 2595, arXiv:1110.0673 [gr-qc].
[11] M. Banados, C. Teitelboim, J. Zanelli, Dimensionally continued black holes, Phys. Rev. D 49 (1994) 975, arXiv:gr-qc/9307033.
[12] J. Crisostomo, R. Troncoso, J. Zanelli, Black hole scan, Phys. Rev. D 62 (2000) 084013, arXiv:hep-th/0003271.
[13] J. Saavedra, R. Troncoso, J. Zanelli, Degenerate dynamical systems, J. Math. Phys. 42 (2001) 4383, arXiv:hep-th/0011231.
[14] M. Bañados, L.J. Garay, M. Henneaux, The local degrees of freedom of higher dimensional pure Chern–Simons theories, Phys. Rev. D 53 (1996) 593, arXiv:hep-th/9506187.
[15] M. Bañados, L.J. Garay, M. Henneaux, The dynamical structure of higher dimensional Chern–Simons theory, Nucl. Phys. B 476 (1996) 611, arXiv:hep-th/9605159.
[16] N. Dadhich, R. Durka, N. Merino, O. Miskovic, Dynamical structure of pure Lovelock gravity, Phys. Rev. D 93 (2016) 064009, arXiv:1511.02541 [hep-th].
[17] G. Giribet, N. Merino, O. Miskovic, J. Zanelli, Black hole solutions in Chern–Simons AdS supergravity, J. High Energy Phys. 1408 (2014) 083, arXiv:1406.3096 [hep-th].
[18] S. Deser, J. Franklin, Canonical analysis and stability of Lanczos–Lovelock gravity, Class. Quantum Gravity 29 (2012) 072001, arXiv:1110.6085 [gr-qc].
[19] P.K. Concha, R. Durka, C. Inostroza, N. Merino, E.K. Rodríguez, Pure Lovelock gravity and Chern–Simons theory, Phys. Rev. D 94 (2016) 024055, arXiv:1603.09424 [hep-th].
[20] J.A. de Azcárraga, J.M. Izquierdo, M. Picón, O. Varela, Generating Lie and gauge free differential (super)algebras by expanding Maurer–Cartan forms and Chern–Simons supergravity, Nucl. Phys. B 662 (2003) 185, arXiv:hep-th/0212347.
[21] F. Izaurieta, E. Rodríguez, P. Salgado, Expanding Lie (super)algebras through Abelian semigroups, J. Math. Phys. 47 (2006) 123512, arXiv:hep-th/0606215.
[22] L. Andrianopoli, N. Merino, F. Nadal, M. Trigiante, General properties of the expansion methods of Lie algebras, J. Phys. A 46 (2013) 365204, arXiv:1308.4832 [gr-qc].
[23] F. Izaurieta, P. Minning, A. Perez, E. Rodríguez, P. Salgado, Standard general relativity from Chern–Simons gravity, Phys. Lett. B 678 (2009) 213, arXiv:0905.2187 [hep-th].
[24] P.K. Concha, D.M. Peñafiel, E.K. Rodríguez, P. Salgado, Even-dimensional general relativity from Born–Infeld gravity, Phys. Lett. B 725 (2013) 419, arXiv:1309.0062 [hep-th].
[25] P.K. Concha, D.M. Peñafiel, E.K. Rodríguez, P. Salgado, Chern–Simons and Born–Infeld gravity theories and Maxwell algebras type, Eur. Phys. J. C 74 (2014) 2741, arXiv:1402.0023 [hep-th].
[26] P.K. Concha, D.M. Peñafiel, E.K. Rodríguez, P. Salgado, Generalized Poincare algebras and Lovelock–Cartan gravity theory, Phys. Lett. B 742 (2015) 310, arXiv:1405.7078 [hep-th].
[27] N. Gonzalez, P. Salgado, G. Rubio, S. Salgado, Einstein–Hilbert action with cosmological term from Chern–Simons gravity, J. Geom. Phys. 86 (2014) 339.
[28] D.V. Soroka, V.A. Soroka, Tensor extension of the Poincaré algebra, Phys. Lett. B 607 (2005) 302, arXiv:hep-th/0410012.
[29] D.V. Soroka, V.A. Soroka, Semi-simple extension of the (super)Poincare algebra, Adv. High Energy Phys. 2009 (2009) 234147, arXiv:hep-th/0605251.
[30] R. Durka, J. Kowalski-Glikman, M. Szczachor, Gauged AdS–Maxwell algebra and gravity, Mod. Phys. Lett. A 26 (2011) 2689, arXiv:1107.4728 [hep-th].
[31] J. Diaz, O. Fierro, F. Izaurieta, N. Merino, E. Rodríguez, P. Salgado, O. Valdivia, A generalized action for (2 + 1)-dimensional Chern–Simons gravity, J. Phys. A 45 (2012) 255207, arXiv:1311.2215 [gr-qc].

[32] P. Salgado, S. Salgado, $\mathfrak{so}(D-1,1) \otimes \mathfrak{so}(D-1,2)$ algebras and gravity, Phys. Lett. B 728 (2013) 5.
[33] P.K. Concha, R. Durka, N. Merino, E.K. Rodríguez, New family of Maxwell like algebras, Phys. Lett. B 759 (2016) 507, arXiv:1601.06443 [hep-th].
[34] R. Durka, Resonant algebras and gravity, arXiv:1605.00059 [hep-th].
[35] R. Schrader, The Maxwell group and the quantum theory of particles in classical homogeneous electromagnetic fields, Fortschr. Phys. 20 (1972) 701.
[36] H. Bacry, P. Combe, J.L. Richard, Group-theoretical analysis of elementary particles in an external electromagnetic field. 1. The relativistic particle in a constant and uniform field, Nuovo Cimento A 67 (1970) 267.
[37] M. Brigante, H. Liu, R.C. Myers, S. Shenker, S. Yaida, The viscosity bound and causality violation, Phys. Rev. Lett. 100 (2008) 191601, arXiv:0802.3318 [hep-th].
[38] L. Aranguiz, O. Miskovic, Free energy of a Lovelock holographic superconductor, Eur. Phys. J. C 74 (2014) 2975, arXiv:1308.2976 [hep-th].
[39] X.O. Camanho, J.D. Edelstein, J.M. Sánchez De Santos, Lovelock theory and the AdS/CFT correspondence, Gen. Relativ. Gravit. 46 (2014) 1637, arXiv:1309.6483 [hep-th].
[40] L. Aranguiz, X.M. Kuang, O. Miskovic, Topological black holes in pure Gauss–Bonnet gravity and phase transitions, Phys. Rev. D 93 (2016) 064039, arXiv:1507.02309 [hep-th].
[41] S. Deser, G.W. Gibbons, Born–Infeld–Einstein actions?, Class. Quantum Gravity 15 (1998) L35, arXiv:hep-th/9803049.
[42] S. Deser, R. Jackiw, S. Templeton, Topologically massive gauge theories, Ann. Phys. 140 (1982) 372.
[43] P.K. Concha, E.K. Rodríguez, Maxwell superalgebras and Abelian semigroup expansion, Nucl. Phys. B 886 (2014) 1128, arXiv:1405.1334 [hep-th].
[44] P.K. Concha, E.K. Rodríguez, P. Salgado, Generalized supersymmetric cosmological term in $N = 1$ supergravity, J. High Energy Phys. 08 (2015) 009, arXiv:1504.01898 [hep-th].
[45] K. Hinterbichler, R.A. Rosen, Interacting spin-2 fields, J. High Energy Phys. 1207 (2012) 047, arXiv:1203.5783 [hep-th].
[46] C. de Rham, A. Matas, A.J. Tolley, New kinetic interactions for massive gravity?, Class. Quantum Gravity 31 (2014) 165004, arXiv:1311.6485 [hep-th].
[47] S.F. Hassan, R.A. Rosen, Bimetric gravity from ghost-free massive gravity, J. High Energy Phys. 1202 (2012) 126, arXiv:1109.3515 [hep-th].
[48] W. Li, Novel nonlinear kinetic terms for gravitons, arXiv:1508.03246 [gr-qc].
[49] X.O. Camanho, J.D. Edelstein, J. Maldacena, A. Zhiboedov, Causality constraints on corrections to the graviton three-point coupling, J. High Energy Phys. 1602 (2016) 020, arXiv:1407.5597 [hep-th].
[50] J.D. Edelstein, M. Hassaine, R. Troncoso, J. Zanelli, Lie-algebra expansions, Chern–Simons theories and the Einstein–Hilbert Lagrangian, Phys. Lett. B 640 (2006) 278, arXiv:hep-th/0605174.
[51] J.A. Azcarraga, K. Kamimura, J. Lukierski, Generalized cosmological term from Maxwell symmetries, Phys. Rev. D 83 (2011) 124036, arXiv:1012.4402 [hep-th].