Self-Organization of Vocabularies under Different Interaction Orders

Javier Vera*
Universidad Adolfo Ibáñez

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Vocabulary, automata networks, interaction orders, naming game

Abstract Traditionally, the formation of vocabularies has been studied by agent-based models (primarily, the naming game) in which random pairs of agents negotiate word-meaning associations at each discrete time step. This article proposes a first approximation to a novel question: To what extent is the negotiation of word-meaning associations influenced by the order in which agents interact? Automata networks provide the adequate mathematical framework to explore this question. Computer simulations suggest that on two-dimensional lattices the typical features of the formation of word-meaning associations are recovered under random schemes that update small fractions of the population at the same time; by contrast, if larger subsets of the population are updated, a periodic behavior may appear.

1 Introduction

To what extent, in a population of language users, is the formation of a word-meaning association (the simplest version of a *vocabulary*) influenced by the order in which agents interact? This problem has been extensively studied within agent-based models, primarily, the *naming game* [14, 15, 3], in which random pairs of agents (one speaker, one hearer) negotiate word-meaning associations at each discrete time step. In order to take into account properties of real social networks [1, 18, 11], recent works have stressed the role of topology in the emergence of a communication system [2, 4, 8, 9]. Nevertheless, in a more realistic scenario it is necessary to consider in turn that in some time frame—for instance, in the same minute—multiple unrelated communicative interactions occur. Models of consensus dynamics thus have to reconsider this new question and to study the influence of various interaction orders on the dynamics.

Automata networks (ANs) [17, 19] provide an adequate mathematical framework to stress the self-organized nature of the formation of vocabularies [14–16, 10]. ANs are extremely simple models where each vertex of a set of vertices (the network) evolves following a local rule based on the states of "nearby" vertices. At each time step, the entire set or a fraction of the set of vertices (even one vertex) is updated. This model is therefore the natural one to describe the influence of the order in which the agents interact on the consensus of the entire population. A previous work [7] has studied theoretically two extreme cases: (1) one vertex is updated at each time step, and (2) all vertices are updated at the same time. A remaining task is then to describe, through computer simulations,

* Facultad de Ingeniería y Ciencias, Universidad Adolfo Ibáñez, Avda. Diagonal Las Torres 2640, Peñalolén, Santiago, Chile. E-mail: jxvera@gmail.com

cases (1) and (2), as well as the intermediate behavior between them, that is, a dynamics where at each time step only a subset of the vertices is updated.

The work is organized as follows. Section 2 explains basic notions and the local rules of the AN model. This is followed by computer simulations focused on the dynamics under four interaction orders. Finally, a brief discussion about the relations between the results and the formation of language is presented.

2 Automata Networks

The agents are located on the vertices of a connected, simple, and undirected graph $\mathcal{G} = (P, I)$, where $P = \{1, \ldots, n\}$ is the set of vertices (the population) and $I = \{1, \ldots, m\}$ is the set of edges. In order to constrain the communicative interactions, the *neighborhood* of the vertex $u \in P$ is defined as the set $V_u = \{v \in P : (u, v) \in I\}$. The vertex u uniquely interacts with its *neighbors*, located in V_u.

A set of p words, W, is considered. Each agent is completely characterized by its *state*, which evolves within communicative interactions. The state associated to the agent $u \in P$ is the pair (M_u, x_u), where $M_u \subseteq W$ is the memory to store words of u, and $x_u \in M_u$ is a word that u conveys to its neighbors of V_u. The set of words conveyed by the neighbors of the vertex u is denoted $W_u = \{x_v : v \in V_u\}$. Since the formation of a language is based only on *local* interactions, the vertex u only receives the words of the set W_u. In principle, it is reasonable to think that the vertex u plays the role of *hearer* (it hears the words conveyed by the neighbors of u), and the vertices of V_u play the role of *speaker* (they convey words to the vertex u). Communicative interactions in the AN model involve multiple speakers (the neighbors) and one hearer (the central vertex).

The AN model is the tuple $\mathcal{A} = (\mathcal{G}, Q, (f_u : u \in P), \phi)$, where

- Q is the set of all possible states of the vertices ($Q = \mathcal{P}(W) \times W$, where $\mathcal{P}(W)$ means the set of subsets of W),
- $(f_u : u \in P)$ is the family of *local rules* (the state of a given vertex u evolves taking into account the states of its neighbors of V_u), and
- ϕ is a function, the *updating scheme*, that gives the order in which the vertices are updated.

The updating schemes can be viewed as different forms of *information transmission* [5, 6]. In the first place, two deterministic schemes are defined. An AN classically follows a *synchronous* scheme, that is, all vertices change their state simultaneously at each discrete time step, and thus the global configuration at a given time is obtained from the entire *information* of the system at the previous time. A second deterministic scheme is the *sequential* one, where the only information relevant for the updating is the order of updating. Put differently, vertices are updated one by one in a prescribed order (a *permutation* of the set of vertices).

Secondly, two probabilistic schemes are defined. The α-*asynchronous* scheme defines a *synchrony* rate: At each time step, each vertex is updated with probability α (particularly, $\alpha = 1$ is equivalent to the synchronous scheme). In this case, only a fraction of the information of the system at a given time is needed to update the system to the next state. Finally, in the *fully asynchronous* scheme vertices are updated one by one in a uniformly random order. Thus, the global configuration at a given time arises from the information contained in only one vertex at the previous time.

A *configuration* $X(t)$ at t is the set of states $\{(M_u(t), x_u(t))\}_{u \in P}$. The configuration $X(t+1)$ at time step $t+1$ is given by the application of local rules $\{f_u\}_{u \in P}$ to a subset of agents defined by the updating scheme. There are two special configurations: (1) a *cycle* (a finite periodic set of configurations), and (2) a *fixed point* (a configuration that is invariant under the application of local rules). Here, a fixed point involves the same final absorbing state of the naming game, in which there is a unique shared word.

3 Two Alignment Strategies Based on the Naming Game

The model proposed here advances the understanding of how a population can reach agreement on which word to use to express a certain meaning. A central aspect of the solution to this problem is *alignment* [16], which means that during communicative interactions language users prefer to use the linguistic items that give the highest chance of agreement based on previous evidence.

In the naming game, agents play the following alignment strategy: (1) a successful word-meaning association (i.e., the speaker and the hearer agree on the association) remains in the agent's inventories, whereas all competing synonyms are canceled; (2) an unsuccessful association instead is stored, as a new synonym, in the hearer's inventory. In the computational model studied here, closely inspired by the naming game, the alignment strategy is *cell-centered*. This means that alignment varies in the following terms: (1′) a successful association (i.e., the speakers and the hearer agree on it) remains in the agent's inventories, whereas all competing synonyms are canceled from the hearer's inventory; (2′) (possibly many) unsuccessful associations are stored, as new synonyms, in the hearer's inventory.

More precisely, the local rule of the AN model involves two alignment strategies, which update the state pair (M_u, x_u) of each vertex $u \in P$:

- the *addition* (**A**) updates M_u by adding words, and
- the *collapse* (**C**) updates M_u by canceling all its words, except one of them.

In the first place, the agents add all the unknown words. This means that the vertex $u \in P$ adds any word (conveyed by its neighbors) $x \in W_u$, so that $x \notin M_u$.

Secondly, in the case that $W_u \subseteq M_u$ (all received words belong to the memory), the constraints of the AN model impose the definition of a mechanism that allows it to *discriminate* between words. The vertex u receives indeed one word from each neighbor of V_u. Discriminating words can be viewed in accordance with the following hypothetical scenario [12]. On a population of early hominids for which leopards represent a higher risk than cows, the word "leopard" may be more valuable than "cow." Given that context, agents prefer therefore the word "leopard," or think it has the higher relevance. In [7], agents prefer, for instance, the most represented association, the *minimal* one (defined—as an interpretation—by its degree of *relevance* [13]) or a randomly picked one.

In this article, each agent is endowed with an internal total order for the set of words (equivalently, the typical order of the set of integers). The agents choose then to converge on the minimum word conveyed in the neighborhood (the minimum of the set W_u).

4 Rules of the Model

Suppose that at time step t the vertex $u \in P$ has been selected according to one of the updating schemes. W_u is the set of all words conveyed by the neighbors of the vertex u. We have $W_u = B_u \cup N_u$, where $B_u = \{x_v \mid (v \in V_u) \wedge (x_v \in M_u)\}$ (the set of *known* words) and $N_u = \{x_v \mid (v \in V_u) \wedge (x_v \notin M_u)\}$ (the set of *unknown* words). The subset $B_u \subseteq W_u$ contains the words conveyed by the neighbors that belong to the memory M_u. In contrast, $N_u \subseteq W_u$ contains the words of W_u that do not belong to M_u. The local rule f_u reads

$$f_u = \begin{cases} \text{if } \emptyset \neq N_u, & (\mathbf{A})(M_i \cup N_u,\, x_u) \\ \text{if } \emptyset = N_u, & (\mathbf{C})(\{\min(B_u)\},\, \min(B_u)) \end{cases}$$

The rule means that in the case that $\emptyset \neq N_u$ (there are words of W_u that the vertex u does not know), the memory M_u is updated by adding the words of N_u. In the other case ($\emptyset = N_u$), the vertex u

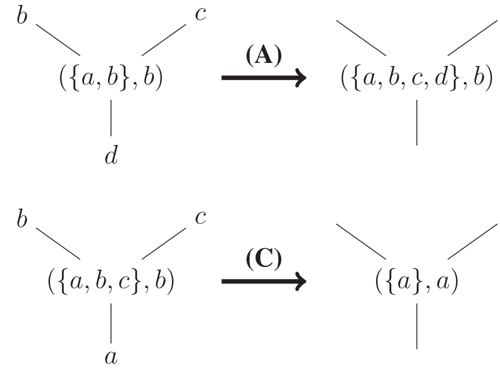

Figure 1. Example of the local rule. All agents share the order $a < b < c < d$. Suppose that at some time step the vertex u has been chosen (the central vertex). Four agents participate of the interaction: the central vertex u and its three neighbors of V_u. Associated with each action of the local rule, two different configurations are shown. For the first row (**A**), $W_u = B_u \cup N_u = \{b\} \cup \{c, d\}$. For the second row (**C**), $W_u = B_u = \{a, b, c\}$.

collapses its memory to the minimum of the set B_u ($B_u = W_u$). Clearly, the conveyed word x_u eventually changes when a collapse occurs (see Figure 1).

The underlying alignment strategies of the AN model suppose that, at each time step, each selected agent plays the role of hearer, whereas the agents connected by an edge to the hearer, the *neighbors*, play the role of speaker. Thus, there are as many speakers as there are neighbors of the hearer. A successful interaction means therefore that the hearer agrees with each speaker. In this case, the hearer selects one association, as the unique remaining variant, and throws away all the other competing synonyms from its inventory. Which association? The minimum of the set B_u.

5 Simulations

5.1 Protocol

To explicitly describe the dynamics of the AN, two macroscopic measures are defined [3]: the total number of words of the system,

$$n_w(t) = \sum_{u \in P} |M_u| \tag{1}$$

where $|M_u|$ is the size of the memory M_u; and the number of different words (or synonyms),

$$n_d(t) = \left| \bigcup_{u \in P} M_u \right| \tag{2}$$

where $\cup_{u \in P} M_u$ represents the union of all sets M_u, $u \in P$.

The simulation protocol is defined by the following elements:

- Averages of $n_w(t)$ and $n_d(t)$ over 100 initial conditions, where each vertex is associated to a different state of the form $(\{x\}, x)$, $x \in W$. Then, $n_w(0) = n_d(0) = n$.
- Four updating schemes: sequential, fully asynchronous, synchronous, and α-asynchronous (with α in $\{0.1, 0.5, 0.9\}$).
- A periodic lattice with a von Neumann neighborhood (four nearest neighbors) with $n = 256^2 = 65536$ vertices for both sequential and fully asynchronous schemes, and $n = 64^2 = 4096$ vertices for both synchronous and α-asynchronous schemes.

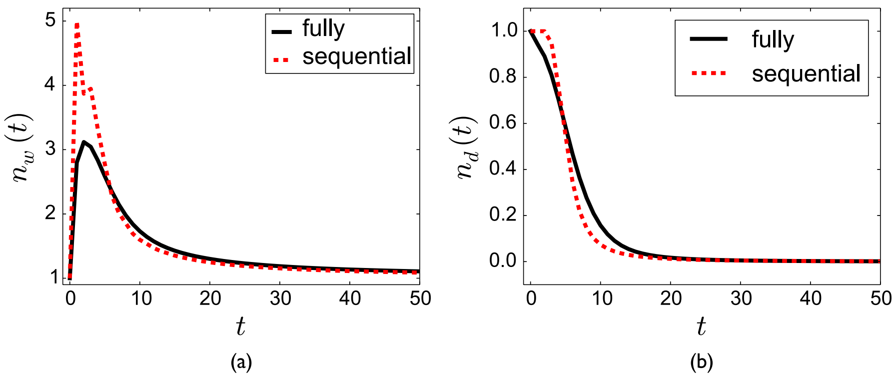

Figure 2. Evolution of $n_w(t)$ and $n_d(t)$ under sequential and fully asynchronous updating schemes. The population is located on an $n = 256^2$ periodic lattice with a von Neumann neighborhood (four nearest neighbors). The simulations run until they reach $n_d(t) = 1$. (a) $n_w(t)$ versus t; (b) $n_d(t)$ versus t. One step t means n vertex updates. The y axis is normalized by n. Only the first 50 steps are shown. Black lines depict the fully asynchronous scheme, whereas red lines depict the sequential scheme. The two schemes converge at $t \approx 300$.

After a finite number of time steps, all simulations, except for the synchronous scheme, reach (or *converge* to) a fixed point at which all vertices share a unique word, that is, $n_w(t) = n$ and $n_d(t) = 1$.

5.2 One Agent Updated at Each Time Step

The dynamics of the AN under a fully asynchronous updating scheme seems to reproduce the typical behavior observed for the naming game on low-dimensional lattices [3, 4], as shown in Figure 2 (black lines). Indeed, the dynamics exhibits three typical domains. First, since at the beginning the agents convey different words, a very fast increase in $n_w(t)$ and a drastic decrease in $n_d(t)$ are observed. Then, a peak in the number of words is reached: $n_w(t) \approx 3n$. Finally, the dynamics enters a very slow convergence to the consensus configuration, where $n_w(t) = n$ and $n_d(t) = 1$. The convergence is reached at $t \approx 300$ steps (only the first 50 steps are shown).

The dynamics under the sequential scheme presents some remarkable aspects, as shown in Figure 2 (red lines). The evolution of the number of words reaches a very sharp peak of $n_w(t) \approx 5n$. This means that at the peak each agent knows all the words conveyed by its neighbors (notice that each vertex has four neighbors). Another interesting feature of the dynamics is that after the peak the dynamics reaches a local maximum at $t \approx 3$. This fact requires further mathematical explanation.

5.3 A Fraction of the (or the Entire) Population Updated at Each Time Step

Synchronous dynamics is exhibited in Figure 3 (blue lines). After approximately 50 time steps (each step means that all agents have been updated), the dynamics enters a periodic behavior with cycles of length 2. Thereby, the number of words, $n_w(t)$, oscillates between n and $2n$, whereas $n_d(t)$ converges to 2. In fact, time steps that correspond to even numbers (even steps) imply collapses ($n_w(t) = n$), and odd steps imply additions ($n_w(t) = 2n$). Small "ladder" steps in the decreasing evolution of $n_d(t)$ show that at odd times only additions are allowed (and then the conveyed words remain fixed).

The dynamics under the α-asynchronous scheme is exhibited in Figure 4. First, at $\alpha = 0.9$ the dynamics presents oscillations that diminish over time until they reach a final consensus fixed point. Second, at $\alpha = 0.1, 0.5$ the dynamics seems to reproduce the behavior observed for the naming game on low-dimensional lattices (like the fully asynchronous scheme). Another interesting result is the presence of scaling in convergence time versus α, as exhibited in Figure 5.

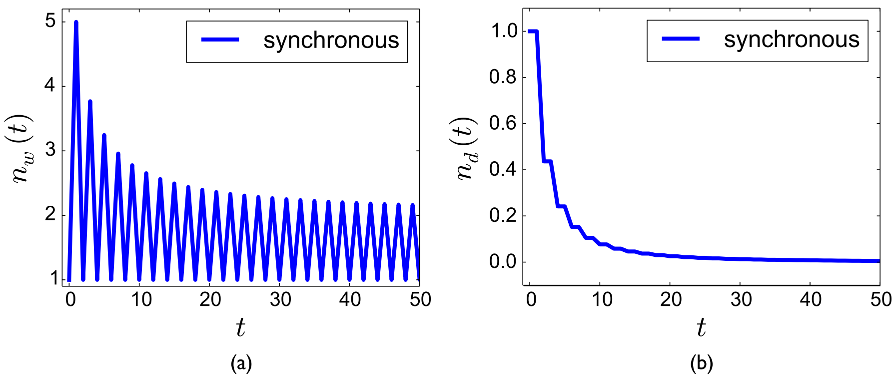

Figure 3. Evolution of $n_w(t)$ and $n_d(t)$ under the synchronous updating scheme. The population is located on an $n = 64^2$ periodic lattice with a von Neumann neighborhood (four nearest neighbors). The simulations run 200 steps. (a) $n_w(t)$ versus t; (b) $n_d(t)$ versus t. One step t means n vertex updates. The y axis is normalized by n. Only the first 50 steps are shown. The dynamics for all initial conditions leads to cycles of size 2.

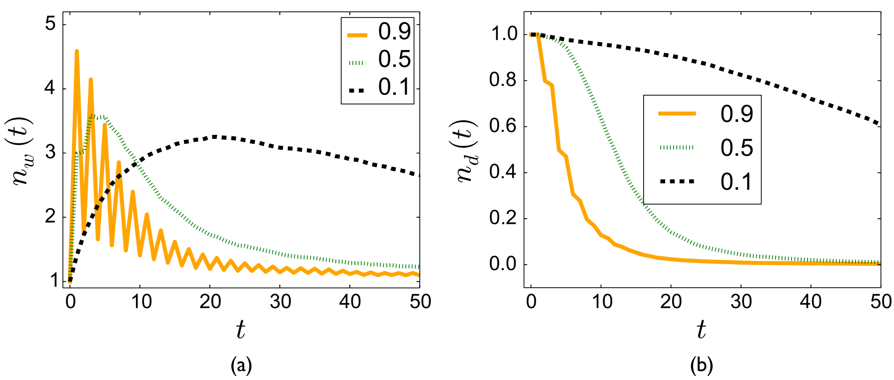

Figure 4. Evolution of $n_w(t)$ and $n_d(t)$ under the α-asynchronous updating scheme. The population is located on an $n = 64^2$ periodic lattice with a von Neumann neighborhood (four nearest neighbors). (a) $n_w(t)$ versus t; (b) $n_d(t)$ versus t. Here α varies among {0.1, 0.5, 0.9}. The y axis is normalized by n. Only the first 50 steps are shown.

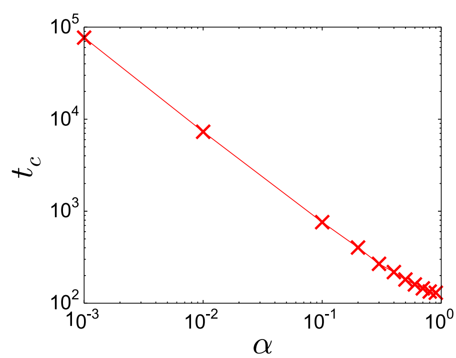

Figure 5. Convergence time t_c versus α, under the α-asynchronous updating scheme. The population is located on an $n = 64^2$ periodic lattice with a von Neumann neighborhood (four nearest neighbors). α varies among {0.001, 0.01, 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9}. Averages over 20 initial conditions (log-log plot).

6 Conclusion

This short article describes computer simulations of a new theoretical framework to study the development of linguistic conventions and, in general, the formation and evolution of language. Despite the fact that the AN model is an *abstract* approximation to the real problem, the work discusses how in a population of agents endowed with simple cognitive mechanisms language arises only from local interactions.

The work proposes two important elements to be discussed: (1) an alternative (mathematical) framework for agent-based studies on language formation, and (2) computer simulations suggesting that on two-dimensional lattices the typical features of the formation of linguistic conventions (as in the naming game) are recovered under random schemes that update small fractions of the population at the same time (fully asynchronous and α-asynchronous, associated with $\alpha \to 0$). In other words, the features of consensus dynamics strongly depend on the amount of *information transmission* between successive time steps.

Many extensions of the proposed model should be studied with the purpose of studying the role of interaction orders on the negotiation of word-meaning associations. In the first place, it seems interesting to explore the dynamics on topologies exhibiting power-law degree distributions. Secondly, a study of *asynchronous information transmission*, understood as disrupting the exchange of information about states between vertices, should be carried out. For instance, in [5] is introduced the β-asynchronous scheme: All vertices are updated simultaneously (as in the synchronous scheme), but the transmission of the new state to the neighborhood depends on a probability β. Finally, the results suggested here should be confirmed by more extensive computer simulations.

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