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Holographic equation of state in fluid/gravity duality

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ABSTRACT

We establish a precise relation between mixed boundary conditions for scalar fields in asymptotically anti de Sitter spacetimes and the equation of state of the dual fluid. We provide a detailed derivation of the relation in the case of five bulk-dimensions for scalar fields saturating the Breitenlohner–Freedman bound. As a concrete example, we discuss the five dimensional scalar-tensor theories describing dark energy in four dimensions.

1. Introduction

It is now widely accepted that there is a precise correspondence between observables in a $(D-1)$-dimensional gauge field theory and a D-dimensional gravity theory. Indeed, since this duality was precisely formulated by Maldacena [1], there has been an increasing amount of conceptual understanding of its meaning. In particular, the long wavelength regime of the duality, known as the fluid/gravity correspondence, has attracted much attention, e.g. [2–5]. It follows from the fact that a relativistic form of the Navier–Stokes equations governing the hydrodynamic limit of a field theory in $(D-1)$ dimensions, on a fixed background γ_{ab}, is equivalent to the dynamics of D-dimensional Einstein gravity with a negative cosmological constant with γ_{ab} as its conformal boundary.

This correspondence allows one to pick a fluid dynamical solution, with an equation of state dictated by the tracelessness of the boundary energy momentum tensor, and reconstruct a bulk solution of the full Einstein equations (for a review see [6]). This opened the possibility of modeling fluid dynamics using general relativity, for instance the elusive description of turbulence has been considered within the fluid/gravity duality. It has been proposed that gravitational dynamics can become turbulent when its dual fluid is at large Reynolds number [7–9]. This has led to the definition of a "gravitational Reynolds number" constructed in terms of the black hole quasinormal modes [10].

In asymptotically anti de Sitter (AdS) spacetimes with a conformally flat boundary, this description is intrinsically limited to traceless energy momentum tensors and so the actual fluid is a very particular one. For understanding realistic fluids by using the fluid/gravity duality, one must be able to describe their dynamics by a general equation of state that is experimentally determined. The holographic relation between the real world systems, which are not conformally invariant in ultraviolet (UV), and gravity requires that the conformal symmetry in the boundary should be broken (see, e.g., [11] for a related discussion on the Wilsonian approach).

The main goal of this Letter is to propose a concrete relation between the equation of state of a (non-conformal) fluid and the asymptotic fall-off behaviour of a scalar field in the AdS bulk. We treat the case of a single scalar field with mass $m^2=-4l^{-2}$, where l is the AdS radius, which is the mass of some of the scalars of type IIB supergravity on $AdS_5\times S^5$. This case is also interesting because the mass saturates the Breitenlohner–Freedman (BF) bound in five dimensions [12,13] and so the logarithmic branches exist [14–16]. We find that, in general, the coefficients of the leading terms in the asymptotic expansion of a scalar field in AdS gravity determine the relationship between the pressure and density of a perfect fluid on the conformal boundary.

This can be traced back to the existence, in any dimension, of two normalizable modes for scalar fields with masses m^2 in the window [17]

$$-\frac{(D-1)^2}{4l^2}=m_{BF}^2\leq m^2<m_{BF}^2+l^{-2}\,. \tag{1}$$

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For our analysis, it is important that they admit mixed boundary conditions, some of which break the conformal invariance at the boundary. This implies that the dual energy momentum tensor is not traceless [18,19] and so the hydrodynamic limit of the field theory is described by a non-conformal fluid. We are going to obtain a general holographic equation of state for a time dependent scalar field using a counterterm method similar in spirit with the one in [19] (the work of [18] is based on the Hamilton–Jacobi equation). It follows that our results are easily generalizable to any dimension and theories with scalars satisfying (1).

Our conventions are defined by the action principle

$$I[g,\phi]=\int_M d^5x\sqrt{-g}\left(\frac{R}{2\kappa}-\frac{1}{2}(\partial\phi)^2-V(\phi)\right)+\frac{1}{\kappa}\int_{\partial M}K\sqrt{-h}+I_{ct}\,, \tag{2}$$

where $\kappa=8\pi G$ is the reduced Newton constant in five dimensions. The potential $V(\phi)$ is required to have at least one local maximum, where it attains a negative value, so that the metric can asymptotically match a locally AdS spacetime. The gravitational counterterms I_{ct} are known from well-established results and render the action principle finite [20,21]. Along the lines of [19] we construct the corresponding scalar counterterms for mixed boundary conditions. We are interested in describing a timelike boundary and so the induced metric on ∂M is $h_{\mu\nu}=g_{\mu\nu}-n_\mu n_\nu$. The extrinsic curvature of the surface with metric $h_{\mu\nu}$ is $K_{\mu\nu}=\frac{1}{2}\mathcal{L}_n h_{\mu\nu}=\frac{1}{2}\left(\nabla_\mu n_\nu+\nabla_\nu n_\mu\right)$ where n_μ is the outwards-pointing normal and $K=h^{\mu\nu}K_{\mu\nu}$. We use below the Einstein equations as defined by the relation

$$E_{\mu\nu}=G_{\mu\nu}-\kappa T_{\mu\nu}=0\,, \tag{3}$$

where $G_{\mu\nu}$ is the Einstein tensor and

$$T_{\mu\nu}=\partial_\mu\phi\partial_\nu\phi-g_{\mu\nu}\left[\frac{1}{2}(\partial\phi)^2+V(\phi)\right]. \tag{4}$$

Working in units where the speed of light and Planck's constant are set to 1, we consider the class of metrics

$$ds^2=-A(t,r)dt^2+B(t,r)dr^2+S(t,r)d\Sigma_k\,, \tag{5}$$

where $d\Sigma_k$ has a constant Ricci scalar $R(\Sigma_k)=6k$, with $k=\pm1$ or 0. The boundary metric is

$$h_{\mu\nu}dx^\mu dx^\nu=-A(t,r)dt^2+S(t,r)d\Sigma_k\,, \tag{6}$$

and the field theory dual metric, which is related by a conformal transformation to $h_{\mu\nu}$, is

$$\gamma_{ab}dx^a dx^b=-dt^2+l^2d\Sigma_k\,. \tag{7}$$

The counterterms I_{ct} are constructed so that the action principle is well-posed and to obtain a finite action. The quasilocal formalism of Brown and York [22] provides a concrete way to compute the action and stress tensor, from which one can directly obtain the mass of the system. We provide an independent calculation of the mass using the Hamiltonian formalism in the next section. In the third section we show that a very precise counterterm exhibits all the desired features reproducing the Hamiltonian result. In the fourth section we compute the dual energy momentum tensor and provide the connection between the equation of state of the dual fluid and the generalized boundary condition of the bulk theory. In the final section we conclude with some comments on these results.

2. Hamiltonian mass

We use the Regge–Teilteiboim approach [23,24] to compute the energy of the system. The main point is that in a theory of gravity, due to the Hamiltonian constraint, the bulk Hamiltonian vanishes and so the conserved charges are associated with the asymptotic symmetries. Therefore, the charges obtained from the Hamiltonian formalism are also appropriate for a holographic interpretation.

We start by noting that the form of the gravitational and scalar contributions to the Hamiltonian is universal when given in terms of its variations

$$\delta H=\delta Q_G+\delta Q_\phi\,, \tag{8}$$

and the concrete expressions can be found in [23] — exact solutions were studied in [25–30]. Let us now consider the case when the scalar field saturates the BF bound, $m^2=-\frac{4}{l^2}$. The fall-off of the scalar field is

$$\phi=\frac{\alpha(t)\ln(r)}{r^2}+\frac{\beta(t)}{r^2}+O\left(\frac{\ln(r)}{r^3}\right). \tag{9}$$

We emphasize that the coefficients α and β can be time-dependent. For some concrete applications when the time-dependence plays an important role see, e.g., [31–34]. In particular, this kind of analysis was useful in describing cosmology models from a holographic point of view [31].

When the metric matches (locally) AdS at infinity the relevant fall-off is

$$g_{tt}=\frac{r^2}{l^2}+k-\frac{\mu(t)}{r^2}+O(r^{-3})\quad,\quad g_{ij}=r^2\Sigma_{ij}+O(r^{-3})\,, \tag{10}$$

$$g_{rr}=\frac{l^2}{r^2}-\frac{l^4k}{r^4}+\frac{l^2}{3}\frac{3M_0(t)l^2+3k^2l^4+\kappa\alpha(t)\left(\alpha(t)-4\beta(t)\right)\ln(r)-2\kappa\alpha(t)^2\ln(r)^2}{r^6}+O\left(\frac{\ln(r)^2}{r^7}\right), \tag{11}$$

where Σ_{ij} is the metric associated with the "angular" part, $d\Sigma_k$.

Inserting these expansions in the Einstein-scalar field equations (3) we find

$$E_t^t-E_r^r=\frac{-12\mu(t)+12M_0(t)+\kappa\alpha^2(t)l^{-2}-4\kappa l^{-2}\alpha(t)\beta(t)+8\kappa l^{-2}\beta^2}{2r^4}+O\left(\frac{\ln(r)^2}{r^5}\right), \tag{12}$$

and so the boundary conditions (10)–(11) are compatible with the field equations provided

$$M_0(t)=\mu(t)-\frac{\kappa l^{-2}}{12}\left(\alpha(t)^2-4\alpha(t)\beta(t)+8\beta(t)^2\right). \tag{13}$$

Using the fall-off of the metric and scalar field we obtain

$$\delta H=\left[\frac{3\delta M_0(t)}{2\kappa}-\frac{1}{l^2}\left(\alpha(t)\delta\beta(t)-2\beta(t)\delta\beta(t)\right)\right]\sigma_k\,, \tag{14}$$

and so the Hamiltonian is finite. Using the relation that arises from the field equations, (13), the result can be written in the form

$$\delta H=\left[\frac{3\delta\mu(t)}{2\kappa}+\frac{1}{l^2}\left(\frac{1}{2}\beta(t)\delta\alpha(t)-\frac{1}{2}\alpha(t)\delta\beta(t)-\frac{\alpha\delta\alpha(t)}{4}\right)\right]\sigma_k\,, \tag{15}$$

and so the Hamiltonian is finite.

To remove the variations from these equations we need to impose boundary conditions on the scalar field. If we write $\beta = \frac{dW(\alpha)}{d\alpha}$ then the right-hand side of (15) is a total variation and the mass of the spacetime is

$$H = \left[\frac{3M_0(t)}{2\kappa} + \frac{1}{l^2}\left(W(\alpha(t)) + \beta(t)^2 - \alpha(t)\beta(t) \right) \right] \sigma_k + H_0 \,, \tag{16}$$

or, using the field equations (13),

$$H = \left[\frac{3\mu(t)}{2\kappa} + \frac{1}{l^2}\left(-\frac{1}{8}\alpha(t)^2 - \frac{1}{2}\alpha(t)\beta(t) + W(\alpha) \right) \right] \sigma_k + H_0 \,, \tag{17}$$

where $\mu(t)$ is the $O(r^{-2})$ coefficient of the g_{tt} and $\delta H_0 = 0$.

There are two cases that yield $H = \frac{3\mu(t)\sigma_k}{2\kappa} + H_0$:

- $\alpha = 0$: these are Dirichlet boundary conditions and ensure asymptotic AdS invariance.
- $\beta = -\frac{1}{2}\alpha \ln(\frac{\alpha}{\alpha_0})$: these are the multi-trace deformations boundary conditions and are compatible with the asymptotic AdS symmetry.

It was originally pointed out in [17] that the evolution of scalar fields in AdS is well defined for Robin boundary conditions for scalar fields with masses that satisfy $m_{BF}^2 \leq m^2 < m_{BF}^2 + l^{-2}$ where m_{BF}^2 is the Breitenlohner–Freedman bound, $m_{BF}^2 = -\frac{4}{l^2}$ [13]. Indeed, it is possible to find this kind of formula in a number of places in the literature [35]. What is new here is that we have taken one order more in the fall off of g_{tt}, namely the μ/r^2 term, and shown how it connects with the standard definition of mass given in terms of the coefficient of $O(r^{-2})$ of g_{rr}^{-1} (see, also, [36]).

3. Counterterm method

First, we restrict our considerations to static configurations. We shall find a counterterm that provides the right free energy in the canonical ensemble for hairy black holes with the fall-off conditions of the previous section. We use the standard technique of Wick rotating the time direction and so the Euclidean path integral yields a thermal partition function. Hence, to study the gauge theory thermodynamics holographically, one has to obtain the action on the Euclidean section.

Due to integration over an infinite volume, the action suffers from infrared divergences that can be regulated by adding suitable boundary terms. With this in mind, the action can be naturally divided into the bulk part I_B, the usual Gibbons–Hawking boundary term I_{GH}, the Balasubramanian–Kraus counterterm I_{BK}, an extrinsic scalar field counterterm, I_{ext}^{ϕ}, and an intrinsic scalar field counterterm, I_{ct}^{ϕ}:

$$\mathrm{i}I^E = I_B + I_{GH} + I_{BK} + I_{ext}^{\phi} + I_{ct}^{\phi} \,, \tag{18}$$

where I^E is the Euclidean action, and at the RHS are the usual Lorentzian expressions integrated in imaginary time $t \in [0, -\mathrm{i}T^{-1})$. The boundary conditions (9)–(11) and field equations (13) yield

$$I_B + I_{GH} + I_{BK} = -\mathrm{i}\frac{\mathcal{A}}{4G} + \mathrm{i}\frac{1}{T}\left[\frac{3M_0}{2\kappa} + \frac{3k^2l^2}{8\kappa} - \alpha^2 l^{-2}\ln(r)^2 + \frac{\alpha l^{-2}}{2}(\alpha - 4\beta)\ln(r) \right]\sigma_k \,, \tag{19}$$

and we see that the logarithmic divergences are proportional to the slower fall-off branch of the scalar field. Here, $\mathcal{A} = \sigma_k S(r_+)^{3/2}$ is the horizon area. It is possible to introduce the following counterterms that provide the correct result for the free energy

$$\begin{aligned} I_{ext}^{\phi} &= \frac{1}{2}\int_{\partial M} d^4x\sqrt{-h}n^{\mu}\phi\partial_{\mu}\phi \\ &= \frac{1}{l^2}\int \sqrt{-\gamma}d^4x\left(-\alpha^2\ln^2 r - \frac{1}{2}\left(\alpha^2 - 4\alpha\beta\right)\ln(r) + \frac{\alpha\beta}{2} - \beta^2 \right) \\ &= -\mathrm{i}\frac{\sigma_k}{l^2T}\left(-\alpha^2\ln^2 r + \frac{1}{2}\left(\alpha^2 - 4\alpha\beta\right)\ln(r) + \frac{\alpha\beta}{2} - \beta^2 \right) \end{aligned} \tag{20}$$

$$I_{ct}^{\phi} = \frac{1}{l^5}\int_{\partial M_{\gamma}} d^4x\sqrt{-\gamma}\left[\frac{\alpha\beta}{2} - W(\alpha) \right] , \tag{21}$$

where we have used the metric γ_{ab} of the dual field theory description. For completeness, we expand at the boundary the RHS of (20) and write it in an explicit intrinsic form, which is an equivalent counterterm written in terms of the local fields on the boundary and a cutoff r along the lines of [37]. The main difference is that we do not use a Fefferman–Graham-like expansion, but the goal is the same, namely to cancel the divergences in the action. The intrinsic counterterm I_{ct}^{ϕ} can be written in terms of the gravity field and a cutoff, r, by using that $\alpha + \frac{\beta}{\ln(r)} + O(\frac{\ln(r)}{r}) = \phi\frac{r^2}{\ln(r)}$ and $\beta = \phi r^2 - \alpha\ln(r)$.

The agreement with the Hamiltonian computation becomes clear if we fix $H_0 = \frac{3k^2l^2}{8\kappa}$ and read off the mass from the Euclidean action

$$I^E = -\frac{\mathcal{A}}{4G} + \frac{1}{T}\left[\frac{3M_0}{2\kappa} + \frac{3k^2l^2}{8\kappa} + \frac{1}{l^2}\left(\beta^2 + W(\alpha) - \alpha\beta \right) \right]\sigma_k \,. \tag{22}$$

Having checked the black hole thermodynamics, we pause to verify that the same counterterms provide a well-posed variational principle, restoring the time dependence. When the field equations hold, the variation of the total action (22) vanishes for Dirichlet boundary conditions for the metric and for scalar field boundary conditions of the form $\beta(t) = \frac{dW}{d\alpha}$, namely

$$\lim_{r\to\infty} \delta I = 0 \,. \tag{23}$$

Let us clarify this further for the scalar field. From (18) we obtain

$$\begin{aligned} \delta I = &\int_M -d^5x\partial_{\mu}\left(\sqrt{-g}g^{\mu\nu}\delta\phi\partial_{\nu}\phi\right) + \frac{1}{2}\int_{\partial M} d^4x\sqrt{-h}n^{\mu}\delta\phi\partial_{\mu}\phi \\ &+ \frac{1}{2}\int_{\partial M} d^4x\sqrt{-h}n^{\mu}\phi\partial_{\mu}\delta\phi \\ &+ \frac{1}{l^5}\int_{\partial M_{\gamma}} d^4x\sqrt{-\gamma}\left(-\frac{\beta(t)}{2} + \frac{\alpha(t)}{2}\frac{d^2W}{d\alpha^2} \right)\delta\alpha(t) \,, \end{aligned} \tag{24}$$

when the field equations hold. Using

$$\begin{aligned} \phi &= \frac{\alpha(t)\ln(r)}{r^2} + \frac{\beta(t)}{r^2} + O(\frac{\ln(r)}{r^3}) \\ \Longrightarrow \delta\phi &= \left(\ln(r) + \frac{d^2W}{d\alpha^2} \right)\frac{\delta\alpha(t)}{r^2} + O(\frac{\ln(r)}{r^3}) \,, \end{aligned} \tag{25}$$

and employing (9)–(11) and (13), it is straightforward to show that (23) indeed holds.

There is one remaining ambiguity in (22), namely that of adding finite counterterms quadratic in the Riemann tensor, Ricci tensor and Ricci scalar of the boundary metric. This is related to the regularization of the field theory dual as discussed in [20].

4. Holographic equation of state

The expectation value of the dual energy–momentum tensor is related to the quasilocal stress tensor (including the counterterms):

$$\langle \mathcal{T}_{ab}\rangle = -\frac{2}{\sqrt{-\gamma}}\frac{\delta I}{\delta \gamma^{ab}} = \lim_{r\to\infty}\frac{r^2}{l^2}\mathcal{T}^{BK}_{\mu\nu} + \lim_{r\to\infty}\frac{r^2}{l^2}\mathcal{T}^{ext}_{\mu\nu} + \mathcal{T}^{ct}_{ab}\,, \tag{26}$$

where the first term is the Balasubramanian–Kraus part [20]

$$\mathcal{T}^{BK}_{\mu\nu} = -\frac{1}{\kappa}\left(K_{\mu\nu} - h_{\mu\nu}K + \frac{3}{l}h_{\mu\nu} - \frac{l}{2}\mathcal{G}_{\mu\nu}\right), \tag{27}$$

with $\mathcal{G}_{\mu\nu}$ the Einstein tensor constructed with the metric h. The second term is the contribution of the extrinsic scalar field term

$$\mathcal{T}^{ext}_{\mu\nu} = \frac{1}{2}h_{\mu\nu}n^{\mu}\phi\partial_{\mu}\phi\,, \tag{28}$$

and the last term is the finite contribution

$$\mathcal{T}^{ct}_{ab} = \frac{1}{l^5}\gamma_{ab}\left[\frac{\alpha(t)\beta(t)}{2} - W(\alpha)\right]. \tag{29}$$

The relevant divergence coming from the bulk and Gibbons–Hawking contributions is canceled out by the divergence from the counterterm and we obtain the following regularized stress tensor of the dual field theory:

$$\begin{aligned}\langle \mathcal{T}_{ab}\rangle = \frac{\gamma_{ab}}{l^3}\Bigg[&-\frac{3M_0(t)}{2\kappa} + \frac{k^2l^2}{8\kappa} + \frac{2\mu(t)}{\kappa} \\ &+ \frac{1}{l^2}\left(\alpha(t)\beta(t) - \beta(t)^2 - W(\alpha)\right)\Bigg] \\ &+ \frac{1}{\kappa l}\delta^0_a\delta^0_b\left[\frac{k^2}{2} + \frac{2\mu(t)}{l^2}\right].\end{aligned} \tag{30}$$

Taking the trace

$$\begin{aligned}\gamma^{ab}\langle \mathcal{T}_{ab}\rangle = \frac{1}{l^3}\Bigg[&-\frac{6M_0(t)}{\kappa} + \frac{6\mu(t)}{\kappa} \\ &+ \frac{4}{l^2}\left(\alpha(t)\beta(t) - \beta(t)^2 - W(\alpha)\right)\Bigg]\end{aligned} \tag{31}$$

and using the field equations (13), we obtain

$$\gamma^{ab}\langle \mathcal{T}_{ab}\rangle = \frac{1}{l^5}\left[\frac{1}{2}\alpha(t)^2 + 2\alpha(t)\beta(t) - 4W(\alpha)\right], \tag{32}$$

which vanishes for the AdS invariant boundary conditions described below equation (17).

Using the normalized timelike vector $u^a = \partial_t$, the energy density of the fluid is

$$\begin{aligned}\rho &= u^a u^b\langle \mathcal{T}_{ab}\rangle \\ &= \frac{1}{l^3}\left[\frac{3M_0(t)}{2\kappa} + \frac{3k^2l^2}{8\kappa} - \frac{1}{l^2}\left(\alpha(t)\beta(t) - \beta(t)^2 - W(\alpha)\right)\right].\end{aligned}$$

The total mass is the energy density integrated on a spacelike section

$$\begin{aligned}M &= \int_\Sigma \rho l^3 d\Sigma \\ &= \left[\frac{3M_0(t)}{2\kappa} + \frac{3k^2l^2}{8\kappa} + \frac{1}{l^2}\left(-\alpha(t)\beta(t) + W(\alpha) + \beta(t)^2\right)\right]\sigma_k \\ &= H\,,\end{aligned} \tag{33}$$

where the last equality is to remark that this result is in agreement with the Hamiltonian computation of the second section. The counterterm computation also provides the Casimir energy of the large N limit of $\mathcal{N}=4$ Super Yang–Mills theory — a cross check of our computation is its exact agreement with the original paper of Balasubramanian–Kraus when the scalar field vanishes [20].

The introduction of the scalar field yields a perfect dual fluid with energy momentum tensor

$$\langle \mathcal{T}_{ab}\rangle = (\rho + p)\,u_a u_b + p\gamma_{ab}\,.$$

Hence, we can identify

$$\begin{aligned}p = \frac{1}{l^3}\Bigg[&\frac{M_0(t)}{2\kappa} + \frac{k^2l^2}{8\kappa} \\ &+ \frac{1}{l^2}\left(\frac{\alpha(t)^2}{6} + \frac{1}{3}\alpha(t)\beta(t) + \frac{1}{3}\beta(t)^2 - W(\alpha)\right)\Bigg],\end{aligned} \tag{34}$$

$$\rho = \frac{1}{l^3}\left[\frac{3M_0(t)}{2\kappa} + \frac{3k^2l^2}{8\kappa} - \frac{1}{l^2}\left(\alpha(t)\beta(t) - \beta(t)^2 - W(\alpha)\right)\right], \tag{35}$$

where we have used the relation (13). Note that when there is no scalar field we get a thermal gas of massless particles $\rho = 3p$ [38]. We remark that, once the boundary conditions are fixed, infrared regularity conditions in the bulk must still be imposed. This finally fixes $M_0 = M_0(\alpha)$, implying there is only one integration constant in the black hole solutions. To fix M_0 as function of α for a given boundary condition is necessary to fix the theory, namely the scalar field potential, and to use standard numerical techniques, see the discussion in [31].

Since $\beta = \frac{dW(\alpha)}{d\alpha}$, the density and pressure are specified by the choice of $W(\alpha)$. This is tantamount to defining an equation of state. Conversely, specification of an equation of state $p = p(\rho)$ necessarily determines $W(\alpha)$ from equations (34) and (35). Indeed, we need $M_0(\alpha)$ to determine the exact equation of state of the perfect fluid. However, we will keep the discussion general and treat a simple case. Using the notation of the second section we can write the pressure and the density as follows:

$$p = \frac{1}{l^3}\left[\frac{\mu(t)}{2\kappa} + \frac{k^2l^2}{8\kappa} + \frac{1}{8l^2}\left(\alpha(t)^2 + 4\alpha(t)\beta(t) - 8W(\alpha)\right)\right], \tag{36}$$

$$\rho = \frac{1}{l^3}\left[\frac{3\mu(t)}{2\kappa} + \frac{3k^2l^2}{8\kappa} - \frac{1}{8l^2}\left(\alpha(t)^2 + 4\alpha(t)\beta(t) - 8W(\alpha)\right)\right], \tag{37}$$

and we shall set the infrared regularity condition by requiring a relation between μ and α. Writing

$$\mu = \alpha^3\frac{d\omega(\alpha)}{d\alpha}\,, \tag{38}$$

we find that the equation of state $p = c_s^2\rho$, where c_s^2 is a constant speed of light squared, is equivalent to the following one-parameter family of boundary conditions

$$W(\alpha) = \frac{l^2\left(3c_s^2 - 1\right)}{\kappa\left(c_s^2 + 1\right)}\left(\alpha^2\omega(\alpha) - \frac{k^2l^2}{8}\right) - \frac{\alpha^2}{4}\ln\left(\frac{\alpha}{\alpha_0}\right), \tag{39}$$

where α_0 is an integration constant. We readily see that when $c_s^2 = \frac{1}{3}$ we recover the description of the gas of massless particles and the AdS invariant boundary conditions discussed in section 2. When the four dimensional fluid is a four dimensional cosmological constant ($c_s^2 = -1$), we find

$$\mu = -\frac{k^2 l^2}{4}, \tag{40}$$

with the function $W(\alpha)$ undetermined. Hence, a four dimensional cosmological constant is special because it fixes the infrared condition but not the boundary condition.

5. Conclusions

We have obtained a concrete and general expression (30) for the dual stress tensor in Einstein-dilaton theories that depends on the boundary conditions imposed on the scalar field; in the context of a perfect-fluid interpretation we obtained explicit expressions (36) for the pressure and (37) for the density. In this way we can model putative dual theories by constructing the equation of state as a function of the Robin boundary conditions on the scalar field. These boundary conditions can, in general, break the conformal symmetry on the boundary; an important advantage of our analysis is that it takes this into account.

We emphasize that our results hold for time dependent bulk configurations; there is no requirement that the fluid respect conformal symmetry. Furthermore, one can choose the scalar field boundary conditions so that the dual fluid describes four-dimensional dark energy. However, a general equation of state can be modeled by choosing appropriate parameters in equation (39).

Although we have worked in five dimensions for scalar fields saturating the Breitenlohner–Freedman bound, our results are easily generalized to any spacetime dimension for any scalar field with masses between this bound and the unitarity bound. This can be of particular use to the holographic description of metals, superconductors and different kind of materials [39]. The holographic description of condensed matter systems has recently been discussed in the hydrodynamic regime [40]. The results brought in here allow to actually introduce a detailed description of the condensed matter system through its equation of state, in the holographic picture.

We point out that our approach (together with information on IR data) was useful to obtaining a universal formula of the speed of sound [41]. The formalism introduced here also has a direct application on the exact, time dependent hairy black hole solutions in Einstein-dilaton gravity with general moduli potential, recently constructed in [42–46] (or the AdS boson stars [47,48]). Indeed, all these collapsing black holes are dual to some process in fluid/gravity with a very precise equation of state that can now be unveiled. It should be noted that the time dependent black holes do not have an straightforward Euclidean continuation, as the naive wick rotation makes the metric complex. The Lorentzian formalism is thus more natural for these configurations.

We also found remarkable the possibility of describing a positive, four dimensional, cosmological constant, using the fluid/gravity correspondence, but a detailed analysis of the holographic properties is beyond the scope of this Letter. However, along the same lines, an equation of state (but for a bulk theory with two scalar fields) was used in [34] to describe some holographic properties of dS spacetime. The fact that the cosmological constant equation of state fixes the infrared regularity condition in such specific form, implies that the five-dimensional scalar-tensor theories allowing for this equation of state, namely admitting the infrared regularity condition (40), should be very peculiar, and probably interesting in their own right.

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