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ORIGINAL PAPER

A stochastic methodology for risk assessment of a large earthquake when a long time has elapsed

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Abstract We propose a stochastic methodology for risk assessment of a large earthquake when a long time has elapsed from the last large seismic event. We state an approximate probability distribution for the occurrence time of the next large earthquake, by knowing that the last large seismic event occurred a long time ago. We prove that, under reasonable conditions, such a distribution is exponential with a rate depending on the asymptotic slope of the cumulative intensity function corresponding to a nonhomogeneous Poisson process. As it is not possible to obtain an empirical cumulative distribution function of the waiting time for the next large earthquake, an estimator of its cumulative distribution function based on existing data is derived. We conduct a simulation study for detecting scenario in which the proposed methodology would perform well. Finally, a real-world data analysis is carried out to illustrate its potential applications, including a homogeneity test for the times between earthquakes.

Keywords Earthquake data analysis · Exponential and gamma distributions · Maximum-likelihood method · Monte Carlo simulation · Nonhomogeneous Poisson process

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1 Introduction

A deep discussion on whether it is possible to predict an earthquake is given by Kagan (1997), but an answer to this question is still open. Throughout the time, some principles have been taken into account to predict the occurrence of earthquakes. Three laws that support these principles are as follows: (i) Omori's law, which sets the rate at which aftershocks occur immediately after a large earthquake or main shock (Omori 1895); (ii) Reid's law, which indicates that an earthquake must have involved an elastic rebound due to accumulated stress (Reid 1910); and (iii) Gutenberg and Richter's law, which establishes a relationship between the magnitude and total number of earthquakes (Gutenberg and Richter 1944). In the latter study, earthquakes have been classified into two groups: The first of them (G1) corresponds to earthquakes of medium intensity fluctuating between 7.0 and 8.4 Richter degrees (°R). The second of these groups (G2) corresponds to large earthquakes with intensity ranging from 8.5 °R with no upper limit.

Time-predictable and slip-predictable models arise from the elastic rebound law and predict the time and slip of the next earthquake, respectively. On the one hand, the time-predictable model assumes that a critical threshold exists, which is constant over time, and once it is attained, an earthquake should occur. On the other hand, the slip-predictable model assumes that a constant minimum stress is present and all the stress accumulated since the last earthquake, over this minimum, is released in the next seismic event. For details about these models, see Shimazaki and Nakata (1980) and Panthi et al. (2011). Other authors, such as Rubinstein et al. (2012), argued that a "memoryless" earthquake model with fixed inter-event time or fixed slip is better than time- and slip-predictable models for earthquake occurrence. However, since according to Kagan (1997) it is not possible to predict earthquakes, we cannot expect that there is homogeneity for the occurrence of them. Anyway, by means of a simple chi-squared test, for a dataset of earthquakes that occurred in the north zone of Chile, we test that the homogeneity hypothesis is not true. In this work, an alternative methodology, which overcomes this lack of homogeneity, is proposed. In the case of censored and missing duration data, a homogeneity test proposed by Balakrishnan and Stehlík (2015) can be used.

The main objective of this work is to propose a stochastic methodology for risk assessment of a large earthquake when a long time has elapsed from the last large seismic event. We state an approximate probability distribution for the waiting time of the next large earthquake, in a specific geological zone, given that a long time has elapsed from the last large earthquake. We prove that, under reasonable conditions, this distribution is exponential with rate depending on the asymptotic slope of the cumulative intensity function (CIF) of a nonhomogeneous Poisson process (NHPP). Our assumptions are quite simple and basically consist in supposing that between the last large earthquake and the next one, $k-1$ $(k \geq 2)$ seismic events of minor intensity occur. Of course, it is not possible to know this parameter k, and any characteristic of the distribution of the waiting time for the next large earthquake only should be based on the data provided by the events of minor intensity. We prove that the limiting distribution of the waiting time for the next large earthquake does not depend on k, whenever a long time is elapsed from the last large earthquake. Moreover, such a limiting distribution is exponential with a parameter depending on the asymptotic slope of the CIF. An estimator for this parameter is provided using existing data from the last large earthquakes. Since it is not possible to obtain an empirical cumulative distribution function (ECDF) of the waiting time for the next large earthquake, this estimator is used to state an estimator of the cumulative distribution function (CDF), which satisfies analogous results to the known theorems of Glivenko–Cantelli and Kolmogorov–Smirnov. For details about these theorems, see Tucker (1967, Sect. 5.4) and Breiman (1968, Sect. 13.6), respectively.

A number of statistical tools have been used in studying seismic activity; see, for example, Kim et al. (2014), Fukutan et al. (2015), and Kamat (2015). Other works on this matter based on point processes are attributed to Ogata (1988) and Adelfio and Chiodi (2015). Indeed, Ogata (1988) modeled seismic activity by means of a Hawkes process and, recently, Fierro (2015) and Fierro et al. (2015) introduced variants of the Hawkes process, which could be more appropriate for modeling earthquakes. Moreover, the asymptotic distribution associated with a shock model based on an NHPP proposed in Fierro et al. (2013) could be adapted for this purpose. However, the methodology proposed in this paper is simpler and easy to use.

The paper is organized as follows: In Sect. 2, we provide some notations and facts on which the results in the later sections rely. Section 3 states conditions for the existence of the limiting CDF, whereas in Sect. 4 an estimator for the asymptotic slope of the CIF associated with an NHPP and its asymptotic properties are studied. In Sect. 5, we prove that this estimator is of maximum-likelihood (ML), and based on it, an estimator of the CDF is defined. Moreover, some properties of this estimator for the limiting CDF are demonstrated. In Sect. 6, a simulation study for detecting the adequacy of the proposed distribution is discussed. In Sect. 7, a real-world data analysis to illustrate the potential applications is carried out. Finally, in Sect. 8, we mention some conclusions of this work and future research.

2 Preliminaries

According to Wesnousky et al. (1984, p. 700), "A correct representation of seismic hazard due to a fault must take into account the time elapsed since the most recent rupture". Based on this quotation, to assess the risk of large earthquakes, one should be focused on determining the probability that the rupture time of a fault (T) occurs during the next h years, conditional on t years elapsed since the last rupture. This conditional probability is given by

$$F_t(h) = \mathbb{P}(t < T \leq t + h | T > t), \quad t > 0, \quad h > 0. \tag{2.1}$$

A number of authors, such as Dargahi-Noubary (1986), Dieterich (1988), and Yakovlev et al. (2007), considered the probability expressed in (2.1) to assess seismic risk. We are also interested in this assessment. From our point of view, it involves stating the probability that a large earthquake occurs, when a long time has elapsed since the last large earthquake. Consequently, a natural way for this assessment consists in assuming that the time S of the next earthquake is a random variable with CDF G satisfying

$$G(h) = \lim_{t \to \infty} F_t(h), \quad h > 0. \tag{2.2}$$

We refer to this function G as the limiting CDF distribution of T. Of course, we must assume that G defined in (2.2) is a CDF. To understand the crucial difference between T and S, we illustrate it with an example. Suppose that T (the recurrence time between earthquakes) follows an Erlang distribution with parameters $r = 2$ and $\lambda = 1$, that is, T has a probability density function (PDF) given by

$$f(t) = \begin{cases} 0, & \text{if} \quad t < 0; \\ t\exp(-t), & \text{if} \quad t \geq 0. \end{cases}$$

Then, from (2.1) and (2.2),

$$F_t(h) = 1 - \exp(-h)\left(\frac{t+h}{t+1}\right)$$

and

$$G(h) = 1 - \exp(-h),$$

respectively. Therefore, the random variable S (the occurrence time of the next large earthquake, knowing a long time without a large seismic event has elapsed) follows an exponential distribution of parameter equal to one.

A realistic model considers only distributions for T such that G corresponds to a nondegenerate distribution. This assumption seems to be quite reasonable, because if a long time has elapsed since the last large earthquake, one should not expect that the next event would occur right now. By assuming this nondegeneracy, we conclude that a number of distributions considered by some authors, such as Gumbel (Cornell 1968; Knopoff and Kagan 1977), lognormal (Nishenko and Buland 1987), and Weibull (Rikitake 1976; Hristopulos and Mouslopoulou 2013), do not seem to be appropriate for modeling the occurrence time of the next seismic event, because each of them leads to degenerate distributions for G given by (2.2). Otherwise, our assumption is not correct or, at least, it contradicts with the use of these distributions.

A possible distribution for T can be obtained as follows: Let $\{T_n\}_{n\in\mathbb{N}}$ be an increasing sequence of occurrence times corresponding to an NHPP with intensity function (IF) λ: $\mathbb{R}_+ \to \mathbb{R}_+$ and associated CIF Λ:$\mathbb{R}_+ \to \mathbb{R}_+$ given by

$$\Lambda(t) = \int_0^t \lambda(u)\,\mathrm{d}u. \tag{2.3}$$

We propose a distribution based on the assumption that some earthquakes of medium and large intensities, in a specific geological zone, occur according to an NHPP with a CIF as given in (2.3). This CIF can be estimated in different ways, as we see later. We assume that the occurrence time of a large earthquake is given by a sum of random variables, each of which represents the time between the occurrence of consecutive earthquakes. Suppose that a large earthquake exactly occurs at the kth shock and this random time is denoted by T_k, the kth jump time of an NHPP. However, as mentioned, it is not possible to know the value of k. We see that our assumptions allow a nondegenerate CDF as defined in (2.2) for S to be obtained, which does not depend on k. The time T_k turns out to have a kind of gamma distribution with parameters depending on k and with CIF Λ defined in (2.3). This distribution is given by means of its PDF in the proposition below (see its proof in "Appendix").

Proposition 2.1 *The PDF of T_k is defined as*

$$f_k(t) = \begin{cases} \dfrac{\lambda(t)}{(k-1)!}\Lambda(t)^{k-1}\exp(-\Lambda(t)), & \text{if} \quad t \geq 0; \\ 0, & \text{if} \quad t < 0, \end{cases} \tag{2.4}$$

where λ is the IF associated with the CIF defined by (2.3).

In the sequel, we assume that λ is a continuous function. Then, the derivative of $\Lambda(t)$ with respect to t is $\lambda(t)$. In addition, as mentioned, we refer to the CDF G, defined in (2.2), as the limiting CDF of T. In the next section, we calculate the limiting CDF of T_k.

3 On the limiting cumulative distribution function

When T follows an exponential distribution, condition (2.2) is trivially satisfied. However, it seems inappropriate to assume a priori that the distribution of T is exponential. The following theorem (see proof in "Appendix") establishes that, under a mild condition, when the limiting CDF of T exists, this corresponds to the exponential distribution or to the degenerate distribution at zero.

Theorem 3.1 *Suppose that T has a continuous PDF f and a limiting CDF G, which is continuously differentiable at zero, from the right, with derivative $G'(0) = m$. Then, for each $h \geq 0$.*

(i) $G(h) = 1 - \lim_{t\to\infty} f(t+h)/f(t)$ and
(ii) $G(h) = 1 - \exp(-mh)$.

Remark 3.1 Let f be as in Theorem 3.1 and suppose that there exists $t_0 > 0$ such that, for each $t \geq t_0$, $f(t) > 0$ and f is decreasing on $[t_0, \infty)$. Hence, from (i) in Theorem 3.1, for each $t \geq t_0$, the function G_t: $\mathbb{R}_+ \to \mathbb{R}_+$ defined as $G_t(h) = 1 - f(t+h)/f(t)$ is a CDF. The family $\{G_t\}_{t>0}$ of CDFs is considered later to conduct some simulations.

Remark 3.2 As in proof of Theorem 3.1 (see "Appendix"), for each $h > 0$, we have that $G'(h) = G'(0)(1 - G(h))$. Thus, $G'(0) > 0$ whenever G is not constant. Hence, in this case, G corresponds to an exponential CDF with parameter $\lambda = G'(0)$. Otherwise, G corresponds to the degenerate distribution at zero.

To illustrate the use of Theorem 3.1, consider the following example.

Example 3.1 Let T be a random variable having a gamma distribution with parameters $\lambda > 0$ and $\beta > 0$, that is, the PDF of T is given by

$$f(t) = \frac{\lambda(\lambda t)^{\beta-1}\exp(-\lambda t)}{\Gamma(\beta)}, \quad t > 0,$$

where Γ is the usual gamma function. Then, T has a limiting CDF given by

$$G(h) = 1 - \exp(-\lambda h), \quad h \geq 0.$$

The next theorem (see proof in "Appendix") provides the limiting CDF of T_k, corresponding to the occurrence time of the kth earthquake, such as defined in Sect. 2.

Theorem 3.2 *Suppose that T has a PDF given by (2.4) and there exists $\lim_{t\to\infty}\lambda(t) = m$, where m is a strictly positive real constant. Then, the limiting CDF of T is given by*

$$G(h) = 1 - \exp(-mh), \quad h \geq 0,$$

with m not depending on k.

Remark 3.3 The constant m in Theorem 3.2 is called the asymptotic slope of Λ.

4 Asymptotic slope of the cumulative intensity

Let $N = \{N_t; t \geq 0\}$ denote an NHPP with CIF Λ defined in (2.3). We assume that the PDF of the occurrence time of the next large earthquake is given by (2.4). Then, it follows from Theorem 3.2 that the asymptotic slope m of Λ, when it exists, is the unique parameter of the limiting CDF, which needs to be estimated. As Λ can be estimated by means of N, the following theorem is the first approach to estimate m.

Theorem 4.1 *Suppose that $\lim_{t\to\infty}\lambda(t) = m$. Then, $\lim_{t\to\infty}\Lambda(t)/t = m$.*

We assume the existence of the asymptotic slope of the CIF to provide more generality to the distribution of T_k. However, the results of this work maintain their importance if we assume that the IF of the NHPP is constant and equal to m. In this case, T_k has an Erlang distribution and its PDF is given by (2.4), replacing $\lambda(t)$ by m. Hence, in practical terms, instead of knowing the asymptotic slope of the CIF, it is enough to know or estimate a constant value of $\lambda(t)$ for all $t \geq \tau^*$, where $\tau^* \geq 0$ is a large enough time instant. The following corollary obtained from Theorem 4.1 aims to this end.

Corollary 4.1 *Let $\tau^* \geq 0$, $m > 0$ and suppose that, for each $t \geq \tau^*$, $\lambda(t) = m$. Then, $\lim_{t\to\infty}(\Lambda(t) - \Lambda(\tau^*))$ $(t - \tau^*) = m$.*

From Corollary 4.1, an estimate of the CIF allows us to estimate its asymptotic slope. Estimates of the CIF for an NHPP have been investigated by a number of authors, such as Arkin and Leemis (2000), Henderson (2003), Leemis (1991, 2004), and Lewis and Shedler (1979). In these works, some estimates are given by the own NHPP. For example, in Henderson (2003), the CIF is estimated in the time interval [0, T), for $T > 0$, as

$$\widehat{\Lambda}(t) = \frac{1}{n}\sum_{i=1}^{n} N_i(0, \ell(t)) + \frac{t - \ell(t)}{\delta}\frac{1}{n}\sum_{i=1}^{n} N_i(\ell(t), \ell(t) + \delta), \quad 0 \leq t < T,$$

where $N_1, \ldots, N_n$ are n independent realizations of an NHPP, $\delta > 0$, $\ell(t) = \lfloor t/\delta \rfloor \delta$, with $\lfloor a \rfloor$ being the floor function or integer part of the number a, and $N_i(a, b)$ denotes the number of events falling in the interval [a, b) for the ith independent realization of the NHPP. The following theorem (see proof in "Appendix") is quite useful to estimate the CIF of our process N.

Theorem 4.2 *Let $\tau^* \geq 0$ and suppose that $\lim_{t\to\infty}\lambda(t) = m$ exists. Then, the following two conditions hold:*

(i) *$\lim_{t\to\infty}(N_t - N_{\tau^*})/(t - \tau^*) = m$, $\mathbb{P}$-almost surely (a.s.) and*

(ii) *$\sqrt{t - \tau^*}((N_t - N_{\tau^*})/(t - \tau^*) - m) \xrightarrow{\mathcal{D}} \mathrm{N}(0, m)$ as $t \to \infty$, whenever $\lim_{t\to\infty}\sqrt{t}(\Lambda(t)/t - m) = 0$, where $\xrightarrow{\mathcal{D}}$ stands for the convergence in distribution and $\mathrm{N}(0, m)$ is a normal random variable with mean zero and variance m.*

Corollary 4.2 *Let $\tau^* \geq 0$ and suppose that $\lambda(t) = m$, for each $t \geq \tau^*$. Then, the family $\{\widehat{m}_t\}_{t>\tau^*}$, defined as $\widehat{m}_t = (N_t - N_{\tau^*})/(t - \tau^*)$, is asymptotically normal distributed with mean m and variance $m/(t - \tau^*)$.*

5 ML estimation and a type of ECDF

The ECDF for the occurrence time of the next large earthquake, conditional on t years elapsed since the last large earthquake, should be defined as

$$\widehat{F}_t^n(h) = \frac{1}{n}\sum_{i=1}^{n} I_{\{R_i \leq h\}}, \tag{5.1}$$

where $R_1, \ldots, R_n$ are n independent random variables having a CDF F_t. However, we cannot observe these random variables, since t is the present time and the evaluation of $\widehat{F_t^n}(h)$ requires the random variables to be observed after time t. Thus, by taking into account that $F_t \to G$ as t goes to ∞, where G is an exponential CDF with parameter m, a natural estimator of G is obtained by means of $\widehat{G}_\tau = 1 - \exp(-\widehat{m}_\tau h)$, where $\widehat{m}_\tau$ is an estimator of m. Note that this fact simplifies significantly the estimation of G when t is large.

Let $\tau^* > 0$ and assume that, for each $t \geq \tau^*$, $\lambda(t) = m$. Then, the CIF of the NHPP has the form

$$\Lambda(t) = \begin{cases} \int_0^t \lambda(s)\mathrm{d}s, & \text{if} \quad 0 \leq t < \tau^*; \\ \int_0^t \lambda(s)\mathrm{d}s + (t - \tau^*)m, & \text{if} \quad \tau^* \leq t. \end{cases}$$

In this section, the parameter m is estimated by means of the ML method. Fix $t > \tau^*$ and let P_t be the distribution of N on the Skorokhod space $\mathrm{D}([0, t], \mathbb{R})$ of the right continuous functions from [0, t] to $\mathbb{R}$, which have left limits. Hence, P_t is absolutely continuous with respect to Q, the distribution of a homogeneous Poisson process (HPP) with rate equal to one. From Theorem 3 in Brémaud (1981) or

Theorem 2.31 in Karr (1991), the Radon–Nikodym derivative of P_t with respect to Q, for $t > \tau^*$ and evaluated at N, is given by

$$\frac{dP_t}{dQ}(N) = \exp\left(\int_0^{\tau^*} \log(\lambda(u))dN_u + \log(m)(N_t - N_{\tau^*}) - \int_0^{\tau^*} (\lambda(u) - 1)du - (t - \tau^*)(m - 1)\right). \quad (5.2)$$

From (5.2), note that the ML estimator of m, for $\tau > \tau^*$, is given by

$$\widehat{m}_\tau = \frac{N_\tau - N_{\tau^*}}{\tau - \tau^*},$$

which coincides with the estimator of m given in Corollary 4.2. Consequently, in the time interval $[0, \tau]$, the limiting CDF G can be determined by the estimator $\widehat{G}_\tau$ of the CDF defined for $h \geq 0$ as $\widehat{G}_\tau(h) = 1 - \exp(-\widehat{m}_\tau h)$. Even though $\widehat{G}_\tau$ is not properly the typical ECDF given in (5.1) based on the independent random variables, a Glivenko–Cantelli type theorem (see Tucker 1967, Sect. 5.4) can be established as given below (see its proof in "Appendix").

Theorem 5.1 *Under assumptions and notations stated in this section, we have that*

$$\lim_{\tau\to\infty} \sup_{h\geq 0} |\widehat{G}_\tau(h) - G(h)| = 0, \quad \mathbb{P}\text{-a.s.}$$

Also, a Kolmogorov–Smirnov type theorem (see Breiman 1968, Sect. 13.6) is given below (see its proof in "Appendix"*).*

Theorem 5.2 *Let $\tau > 0$. Under assumptions and notations stated in this section, we have that*

$$\sup_{h\geq 0} |\sqrt{\tau}(\widehat{G}_\tau(h) - G(h))| \xrightarrow[\tau\to\infty]{\mathcal{D}} \left|\mathrm{N}\left(0, \frac{\exp(-2)}{m}\right)\right|,$$

where $\mathrm{N}(0, \exp(-2)/m)$ is a normal random variable with mean zero and variance $\exp(-2)/m$.

6 Simulation

We have stated the distribution of the occurrence time for a large earthquake, in a specific geological zone, given that a long time has elapsed since the last large seismic event. Then, to detect the scenario which is suitable for the stated distribution to perform well, we conduct the following simulation study with the objective to determine what value of the time can be considered as large enough.

Let $\{G_t\}_{t>0}$ be the family of CDFs defined in Remark 3.1 and G the corresponding limiting CDF with $m = G'(0) > 0$. To evaluate the accuracy of the approximation of G_t by means of G, we simulate n data from a random variable S with CDF G_t for different values of $t \geq 0$. By assuming that the distribution of T_k is given by (2.4) with $\Lambda(t) = mt$ and $m > 0$, we have G_t is expressed as

$$G_t(h) = 1 - \left(1 + \frac{h}{t}\right)^{k-1} \exp(-mh), \quad t \geq 0, \quad h \geq 0. \quad (6.1)$$

Note that, for each $t \geq k - 1$, G_t given in (6.1) is a CDF. Our simulation study consists in (i) partitioning the positive part of the real straight line in r subintervals, which are determined as $0 = h_0 < h_1 < \cdots < h_{r-1} < h_r = \infty$; (ii) determining the observed percentage of times that the n simulated values of S fall into each subinterval $[h_{i-1}, h_i[$; and (iii) computing the corresponding expected percentage given by $100 \times (G(h_i) - G(h_{i-1}))$, for $i = 1, \ldots, r - 1$, and by $100 \times (1 - G(h_{r-1}))$, for $i = r$. From the probability integral transform, $G(S)$ follows a uniform distribution in the interval [0, 1]. Hence, from (i) in Theorem 3.1, we expect that, for large enough values of t, $G_t(S)$ approximately follows a uniform distribution. The simulation is carried out with $n = 1000$ and $r = 10$ class intervals, because it is coherent with the Sturges rule; see Sturges (1926). This rule indicates that a suitable number of class intervals are $r = 1 + \log_2(n)$, which in our case are $r = 1 + \log_2(1000) \approx 10$. We expect that, as t increases, percentages of simulated and expected values must be similar. A goodness-of-fit χ^2 test is used to evaluate this similarity. Indeed, based on such a test, our objective is to determinate the values of t for which the approximation provided in Theorem 3.2 is satisfactory. Specifically, the scenario of the simulation study considers $m = 1$ and $k = 10$. The random variable S is simulated 1000 times, with $r = 10$, where $h_1, \ldots, h_9$ are chosen in such a way that $G(h_i) = i/10$, that is, $h_1 = 0.1053605, h_2 = 0.2231436$, $h_3 = 0.3566749$, $h_4 = 0.5108256, h_5 = 0.6931472, h_6 = 0.9162907, h_7 = 1.2039728$, $h_8 = 1.6094379, h_9 = 2.3025851$. Then, the observed percentages falling into these subintervals are determined; see Table 1.

The expected percentages are all 10 %, which are evaluated using the χ^2 test. For this purpose, we define the statistic $\chi^2 = \sum_{j=1}^{10}(O_j - 10)^2/10 \sim \chi^2(9)$, where, for $j = 1, \ldots, 10$, O_j is the observed value of S falling in the jth interval. For each $i \in \{1, \ldots, 6\}$, we define

$$\chi_i^2 = \sum_{j=1}^{10} \frac{(O_{ij} - 10)^2}{10},$$

where O_{ij} are the values given in Table 1. Thus, the χ^2 test allows us to evaluate the goodness-of-fit of G_t by means of the limiting CDF G; see Table 1. Indeed, the p-value, corresponding to the i-th row of Table 1, is calculated by

Table 1 Percentages of simulated values of S for the indicated time t and p-values of the corresponding χ^2 test ($m = 1$ and $k = 10$)

t	h										
	$[h_0, h_1]$	$[h_1, h_2]$	$[h_2, h_3]$	$[h_3, h_4]$	$[h_4, h_5]$	$[h_5, h_6]$	$[h_6, h_7]$	$[h_7, h_8]$	$[h_8, h_9]$	$[h_9, h_{10}]$	p-value
10	1.1	1.7	1.2	2.0	1.4	3.4	4.9	7.4	12.0	64.9	<0.001
20	5.0	6.3	6.8	6.4	6.8	9.2	10.6	10.1	13.4	25.4	<0.001
25	6.7	6.4	7.0	8.3	7.4	9.2	10.1	11.8	12.5	20.6	0.057
30	7.5	9.0	8.4	7.4	7.7	7.5	9.9	10.1	13.0	19.5	0.175
40	7.3	8.9	9.4	9.2	9.6	8.7	11.7	8.2	11.1	15.9	0.803
50	10.3	9.7	8.2	8.8	9.3	11.5	8.9	9.7	11.1	12.5	0.996

means of the chi-squared statistic χ_i^2. From this table, we conclude that, for $t \geq 25$, the distribution of G_t is well approximated by G.

7 Data analysis

Next, we provide a real-world data analysis to illustrate the potential applications of the proposed methodology. We consider 39 earthquakes registered between the years 1604 and 2007 in the north zone of Chile, with magnitudes between 7.0 and 8.9 °R and their respective dates of occurrence are described in Table 2. In this table, 36 and 3 earthquakes belonging to G1 and G2, respectively, are detected, where an asterisk indicates a large earthquake according to G2. The times when any of the three large earthquakes from G2 occur are considered as zero, marking the beginning of the next period. The recurrence times of earthquakes from G1 until an earthquake from G2 occurs are shown in Table 3, which we call first, second, and third data subsets. The first data subset is not considered in this analysis because it contains too few data; see Table 3. The corresponding estimated asymptotic slope of the CIF $\widehat{m}_t$ for each of these data subsets is also given in Table 3. For $t = 53, 116, 118, 121$ provided in Table 3 (second data subset), the type of ECDF $\widehat{F}_t$ associated with F_t is given in Table 4. In this table, we compare numerically this type of ECDF with the corresponding estimator of the CDF.

In general, homogeneity for the occurrence of earthquakes should not hold. In our application, under homogeneity conditions, during the 405 years in which 39 seismic events were observed, it is expected that 13 earthquakes occur in each period of 135 years. To test homogeneity, once again, we define the statistic $\chi^2 = \sum_{j=1}^{3}(O_j - 13)^2/13 \sim \chi^2(2)$, where, for $j = 1, 2, 3$, O_j is the observed frequency of earthquakes in the jth period. Table 5 presents the observed and expected (E_j) frequencies of earthquakes occurred in each period. From this table, χ^2 statistic is calculated and since $\chi^2 = 26.31 > 9.21 = \chi^2_{0.99}(2)$, we reject the homogeneity hypothesis at 1 % significance level.

As mentioned, because we are unable to known when the next earthquake will occur, it is not possible to calculate an ECDF associated with any F_t. However, from Theorems 5.1 and 5.2, the estimator of the CDF $\widehat{G}_t$ permits us to estimate this CDF. In addition, as illustrated next, Corollary 4.2 allows us to find approximate confidence bands for the unknown limiting CDF G. Let

Table 2 Year and Richter degree for data of earthquakes in the north zone of Chile

Year	1604	1615	1681	1715	1768	1831	1833	1836	1868	1870
°R	8.5*	7.5	7.4	8.8*	7.7	7.6	7.8	7.5	8.0	7.5
Year	1871	1876	1877	1878	1905	1906	1906	1909	1911	1925
°R	7.5	7.2	8.6*	7.3	7.0	7.2	7.0	7.6	7.3	7.3
Year	1928	1933	1936	1940	1945	1947	1948	1953	1956	1965
°R	7.1	7.6	7.3	7.3	7.2	7.0	7.0	7.5	7.1	7.1
Year	1966	1967	1970	1983	1987	1988	1995	2005	2007	→
°R	7.9	7.5	7.0	7.4	7.2	7.0	7.6	7.9	7.6	

Table 3 Subsets obtained from the data of earthquakes in the north zone of Chile with t expressed in years

First data subset				Second data subset								
t	0	11	77	0	53	116	118	121	153	155	156	161
$\widehat{m}_t$	0	$\frac{1}{11}$	$\frac{2}{77}$	0	$\frac{1}{53}$	$\frac{2}{116}$	$\frac{3}{118}$	$\frac{4}{121}$	$\frac{5}{153}$	$\frac{6}{155}$	$\frac{7}{156}$	$\frac{8}{161}$
°R	8.5*	7.5	7.4	8.8*	7.7	7.6	7.8	7.5	8.0	7.5	7.5	7.2

Third data subset

t	0	1	28	29	29	32	34	48	51	56	59	63	68	70
$\widehat{m}_t$	0	1	$\frac{2}{28}$	$\frac{3}{29}$	$\frac{4}{29}$	$\frac{5}{32}$	$\frac{6}{34}$	$\frac{7}{48}$	$\frac{8}{51}$	$\frac{9}{56}$	$\frac{10}{59}$	$\frac{11}{63}$	$\frac{12}{68}$	$\frac{13}{70}$
°R	8.6*	7.3	7.0	7.2	7.0	7.6	7.3	7.3	7.1	7.6	7.3	7.3	7.2	7.0

Third data subset (continuation)

t	71	76	79	88	89	90	93	106	110	111	118	128	130	→
$\widehat{m}_t$	$\frac{14}{71}$	$\frac{15}{76}$	$\frac{16}{79}$	$\frac{17}{88}$	$\frac{18}{89}$	$\frac{19}{90}$	$\frac{20}{93}$	$\frac{21}{106}$	$\frac{22}{110}$	$\frac{23}{111}$	$\frac{24}{118}$	$\frac{25}{128}$	$\frac{26}{130}$	
°R	7.0	7.5	7.1	7.1	7.9	7.5	7.0	7.4	7.2	7.0	7.6	7.9	7.6	

Table 4 Values of ECDF and estimated CDF and their differences for the indicated time with Chilean earthquakes data

h	0	63	65	68	100	102	103	108	109
$\widehat{F}_{53}(h)$	0/8	1/8	2/8	3/8	4/8	5/8	6/8	7/8	8/8
$\widehat{G}_{53}(h)$	0	0.70	0.71	0.72	0.85	0.86	0.86	0.87	0.88
$\lvert\widehat{G}_{53}(h)-\widehat{F}_{53}(h)\rvert$	0	0.57	0.45	0.35	0.35	0.24	0.11	0.01	0.12

h	0	2	5	37	39	40	45	46
$\widehat{F}_{116}(h)$	0/7	1/7	2/7	3/7	4/7	5/7	6/7	7/7
$\widehat{G}_{116}(h)$	0	0.66	0.67	0.69	0.82	0.83	0.83	0.84
$\lvert\widehat{G}_{116}(h)-\widehat{F}_{116}(h)\rvert$	0	0.52	0.39	0.26	0.25	0.03	0.16	

h	0	3	35	37	38	43	44
$\widehat{F}_{118}(h)$	0/6	1/6	2/6	3/6	4/6	5/6	6/6
$\widehat{G}_{118}(h)$	0	0.80	0.81	0.82	0.92	0.93	0.93
$\lvert\widehat{G}_{118}(h)-\widehat{F}_{118}(h)\rvert$	0	0.63	0.48	0.32	0.25	0.09	0.07

h	0	32	34	35	40	41
$\widehat{F}_{121}(h)$	0/5	1/5	2/5	3/5	4/5	5/5
$\widehat{G}_{121}(h)$	0	0.88	0.88	0.89	0.96	0.97
$\lvert\widehat{G}_{121}(h)-\widehat{F}_{121}(h)\rvert$	0	0.68	0.48	0.29	0.16	0.03

$$[m_{t,\alpha}^{-}, m_{t,\alpha}^{+}] = \frac{1}{2}\left(\frac{x_\alpha^2}{t} + 2\widehat{m}_t \pm \frac{x_\alpha}{\sqrt{t}}\sqrt{\frac{x_\alpha^2}{t} + 4\widehat{m}_t}\right), \quad 0<\alpha<1, \tag{7.1}$$

be a $(1-\alpha)\times 100\,\%$ confidence interval for m, with t fixed. Hence, by choosing x_α such that $\mathbb{P}(|\sqrt{t/m}(\widehat{m}_t - m)| \le x_\alpha) = 1-\alpha$, we have $[m_{t,\alpha}^{-}, m_{t,\alpha}^{+}]$ given in (7.1) is a $(1-\alpha)\times 100\,\%$ confidence interval for m, with t fixed. Consequently, $\mathbb{P}\left(\bigcap_{h>0}\{G_{t,\alpha}^{-}(h) \le G(h) \le G_{t,\alpha}^{+}(h)\}\right) = 1-\alpha$, where $G_{t,\alpha}^{-}(h)$ and $G_{t,\alpha}^{+}(h)$ are the CDFs corresponding to the exponential distribution with parameters $m_{t,\alpha}^{-}$ and $m_{t,\alpha}^{+}$, respectively, that is, $G_{t,\alpha}^{-}(h) = 1-\exp(-m_{t,\alpha}^{-}h)$ and $G_{t,\alpha}^{+}(h) = 1-\exp(-m_{t,\alpha}^{+}h)$, for $h \ge 0$. Note that, since $G_{t,\alpha}^{-}$, $G_{t,\alpha}^{+}$, and G are continuous CDFs, the inequalities $G_{t,\alpha}^{-} \le G \le G_{t,\alpha}^{+}$ explain why the bands become more narrow at the extremes. Indeed, on the one hand, we have $G_{t,\alpha}^{-}(0) = G_{t,\alpha}^{+}(0) = 0$ and by continuity, the difference $G_{t,\alpha}^{+}(h) - G_{t,\alpha}^{-}(h)$ should be small if h is small. On the other hand, $\lim_{h\to\infty} G_{t,\alpha}^{-}(h) = \lim_{h\to\infty} G_{t,\alpha}^{+}(h) = 1$ and by continuity again, the difference $G_{t,\alpha}^{+}(h) - G_{t,\alpha}^{-}(h)$ should be

Table 5 Frequency of earthquakes in the indicated period for Chilean earthquakes data

Frequency	Period (years)		
	1601–1736	1737–1872	1873–2008
O_i	4	7	28
E_i	13	13	13

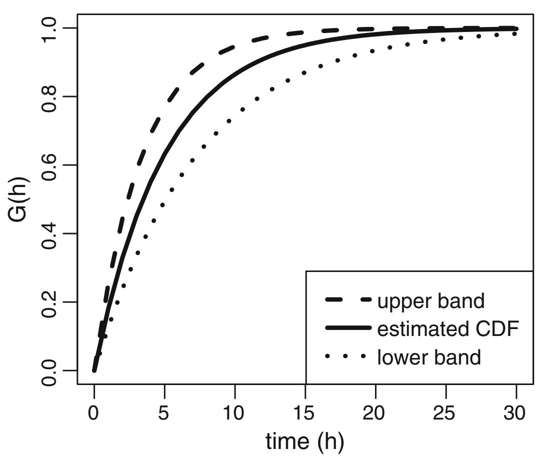

Fig. 1 95 % confidence bands for the CDF G with $t = 130$ for Chilean earthquake data

small for large values of h. Thus, $G^-_{t,\alpha}$ (lower band) and $G^+_{t,\alpha}$ (upper band) are $100 \times (1-\alpha)\,\%$ confidence bands for the limiting CDF G over $h > 0$, with t fixed. From Corollary 4.2, for $\alpha = 0.05$, we have $x_\alpha = 1.96$. For this value of α, $t = 130$ and the data in Table 3 (third data subset), we plot 95 % confidence bands for the CDF G in Fig. 1, from which it is possible to note that, within 10 years more, one has a high probability of occurrence for an earthquake, whereas this probability is practically one within 20 years more.

8 Conclusions and future research

In this paper, we proposed a stochastic methodology for the risk assessment of a large earthquake when a long time has elapsed from the last large seismic event. We proved that, by assuming that earthquakes occur after approximately $k-1$ seismic events of less intensity, the distribution of the occurrence time for the next large earthquake, by knowing that the last large seismic event occurred a long time ago, is exponential with rate depending on the asymptotic slope of the cumulative intensity function. This function corresponds to a nonhomogeneous Poisson process, which does not depend on k. We conclude that, for large values of τ, it is advisable to estimate m, the asymptotic slope of the cumulative intensity function, by $\widehat{m}_\tau = N_\tau/\tau$, where N_τ corresponds to the number of seismic events occurred in $[0,\tau]$. This estimator is consistent for m and, in a number of cases, it turns out be a maximum likelihood estimator. Simulations carried out for $m = 1$, $k = 10$, and p $= 0.057$ suggested that, for $\tau \geq 25$ years, an estimator of the cumulative distribution function $\widehat{G}_\tau$ is a good approximation for G, its limiting function. Because it is not possible to know when the next earthquake will occur, the empirical cumulative distribution function of the waiting time for the next earthquake cannot be evaluated. Finally, a real-world data analysis enabled us to illustrate the potential applications of our proposal. Moreover, for these data, the homogeneity hypothesis for the times between earthquakes was rejected. This fact showed that our methodology is a good alternative for this lack of homogeneity.

We focused our research on the occurrence time of the next earthquake for a specific geological zone. However, the seismic activity in a determined region depends on the adjacent geography, which was not considered here. Consequently, it is natural to ask for the distribution of the random vector consisting of the occurrence times of the next large earthquake for each of the adjacent locations, given that a long time has elapsed since the last large earthquake in the different neighborhood locations. Probably, this is a complex problem, but it could be addressed in a future work. Some studies relating spatial dependency and occurrence of earthquakes are attributed to Adelfio and Chiodi (2015), Nicolis et al. (2015), and references therein.

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Appendix

Proof of Proposition 2.1 Let L and L^λ be an HPP and an NHPP, respectively, where L has rate equal to one and L^λ has intensity λ. Let τ_k and τ^λ_k be the kth jump of L and L^λ, respectively. Consequently, T_k and τ^λ_k are equal in distribution and since L^λ_t has the same distribution that $L_{\Lambda(t)}$, we have

$$\begin{aligned}\mathbb{P}(T_k \leq t) &= \mathbb{P}(\tau^\lambda_k \leq t) = \mathbb{P}(L^\lambda_t \geq k) = \mathbb{P}(L_{\Lambda(t)} \geq k)\\ &= \mathbb{P}(\tau_k \leq \Lambda(t))\\ &= \int_0^{\Lambda(t)} \frac{1}{(k-1)!} u^{k-1} \exp(-u)\mathrm{d}u\\ &= \int_0^t \frac{\lambda(r)}{(k-1)!} \Lambda(r)^{k-1} \exp(-\Lambda(r))\mathrm{d}r,\\ &\quad (\text{by making } u = \Lambda(r)).\end{aligned}$$

Therefore, the PDF of T_k is given by (2.4). □

Proof of Theorem 3.1 Let F be the CDF of T. Then, by the L'Hôpital rule and the fundamental theorem of calculus, we have

$$G(h) = \lim_{t\to\infty} \mathbb{P}(t < T \le t+h | T > t) = 1 - \lim_{t\to\infty} \frac{1-F(t+h)}{1-F(t)} = 1 - \lim_{t\to\infty} \frac{F'(t+h)}{F'(t)} = 1 - \lim_{t\to\infty} \frac{f(t+h)}{f(t)}.$$

Consequently,

$$\frac{G(h+\Delta h)-G(h)}{\Delta h} = \frac{1}{\Delta h}\left(\lim_{t\to\infty} \frac{(f(t+h)-f(t+h+\Delta h))}{f(t+h)} \frac{f(t+h)}{f(t)}\right) = \frac{1}{\Delta h}\left(1 - \lim_{t\to\infty} \frac{f(t+h+\Delta h)}{f(t+h)}\right) \lim_{t\to\infty} \frac{f(t+h)}{f(t)} = \frac{G(\Delta h)(1-G(h))}{\Delta h}. \tag{8.1}$$

Thus, by taking limit as $\Delta h \to 0$ in (8.1), we have that $\mathrm{d}G(h)/\mathrm{d}h = G'(h) = G'(0)(1-G(h))$. This differential equation has a unique solution, for $G'(0) = m$, which is given by $G(h) = 1 - \exp(-mh)$. Therefore, the proof is complete. □

Proof of Theorem 3.2 From the L'Hôpital rule, we have

$$\lim_{t\to\infty} \frac{\Lambda(t+h)}{\Lambda(t)} = \lim_{t\to\infty} \frac{\lambda(t+h)}{\lambda(t)} = 1.$$

In addition, from the mean value theorem, there exists $\xi(t)$ between t and $t+h$, with $t > 0$ and $h \ge 0$, such that $\Lambda(t+h) - \Lambda(t) = \lambda(\xi(t))h$. Consequently,

$$\frac{f_k(t+h)}{f_k(t)} = \exp(-\lambda(\xi(t))h) \left(\frac{\Lambda(t+h)}{\Lambda(t)}\right)^{k-1} \frac{\lambda(t+h)}{\lambda(t)}.$$

Thus, because

$$\lim_{t\to\infty} \frac{f_k(t+h)}{f_k(t)} = \lim_{t\to\infty} \exp(-\lambda(\xi(t))h) = \exp(-mh),$$

the proof follows from (i) in Theorem 3.1. □

Proof of Theorem 4.1 For each $\varepsilon > 0$, let $t_0 > 0$, such that $|\lambda(s) - m| < \varepsilon$, whenever $s \ge t_0$. Consequently, for each $t > t_0$, we have

$$\left|\frac{1}{t}\int_0^t \lambda(s)\mathrm{d}s - m\right| = \left|\frac{1}{t}\int_0^t (\lambda(s)-m)\mathrm{d}s\right| \le \frac{1}{t}\int_0^t |\lambda(s)-m|\mathrm{d}s \le \frac{1}{t}\int_0^{t_0} |\lambda(s)-m|\mathrm{d}s + \frac{1}{t}\int_{t_0}^t |\lambda(s)-m|\mathrm{d}s < \frac{1}{t}\int_0^{t_0} |\lambda(s)-m|\mathrm{d}s + \varepsilon.$$

By taking limit as $t \to \infty$, we obtain

$$\limsup_{t\to\infty} \left|\frac{1}{t}\int_0^t \lambda(s)\mathrm{d}s - m\right| \le \varepsilon.$$

As $\varepsilon > 0$ is arbitrary, we have $\lim_{t\to\infty} \Lambda(t)/t = m$, which concludes the proof. □

Proof of Theorem 4.2 Let $M = \{M_t; t \ge 0\}$ be the martingale defined by $M_t = N_t - \Lambda(t)$. As

$$\frac{N_t - N_{\tau^*}}{t-\tau^*} = \frac{\Lambda(t)-\Lambda(\tau^*)}{t-\tau^*}\left(\frac{M_t - M_{\tau^*}}{\Lambda(t)-\Lambda(\tau^*)} + 1\right),$$

and Theorem 8.2.17 in Dacunha-Castelle and Duflo (1986) implies $\lim_{t\to\infty}(M_t - M_{\tau^*})/(\Lambda(t)-\Lambda(\tau^*)) = 0$, $\mathbb{P}$-a.s., from Theorem 4.1, we have $\lim_{n\to\infty}(N_t - N_{\tau^*})/(t-\tau^*) = m$, $\mathbb{P}$-a.s.

Suppose that $\lim_{t\to\infty}\sqrt{t}(\Lambda(t)/t - m) = 0$ and let L be an HPP with rate equal to one. For each $t \ge \tau^*$, we have

$$\sqrt{t-\tau^*}\left(\frac{L_{\Lambda(t)} - L_{\Lambda(\tau^*)}}{t-\tau^*} - m\right) = \left(\frac{\Lambda(t)}{t-\tau^*}\right)^{1/2}\left(\frac{L_{\Lambda(t)} - \Lambda(t)}{\sqrt{\Lambda(t)}}\right) + \sqrt{t-\tau^*}\left(\frac{\Lambda(t) - L_{\Lambda(\tau^*)}}{t-\tau^*} - m\right).$$

Consequently, to prove that

$$\sqrt{t-\tau^*}\left(\frac{L_{\Lambda(t)} - L_{\Lambda(\tau^*)}}{t-\tau^*} - m\right) \xrightarrow{\mathcal{D}} \mathrm{N}(0,m), \quad \text{as} \quad t\to\infty, \tag{8.2}$$

and since $\lim_{t\to\infty} L_{\Lambda(\tau^*)}/\sqrt{t-\tau^*} = 0$, we need to prove

$$\left(\frac{\Lambda(t)}{t-\tau^*}\right)^{1/2}\left(\frac{L_{\Lambda(t)} - \Lambda(t)}{\sqrt{\Lambda(t)}}\right) \xrightarrow{\mathcal{D}} \mathrm{N}(0,m), \quad \text{as} \quad t\to\infty, \tag{8.3}$$

and

$$\lim_{t\to\infty} \sqrt{t-\tau^*}\left(\frac{\Lambda(t)}{t-\tau^*} - m\right) = 0. \tag{8.4}$$

As $(L_t - t)/\sqrt{t} \xrightarrow{\mathcal{D}} \mathrm{N}(0,1)$ and $\lim_{t\to\infty}\Lambda(t)/(t-\tau^*) = m$, condition (8.3) follows. From the intermediate value theorem, for each $t > 0$, there exists $\xi(t)$ between $t-\tau^*$ and t such that $\Lambda(t) = \Lambda(t-\tau^*) + \lambda(\xi(t))\tau^*$. This fact implies that

$$\sqrt{t-\tau^*}\left(\frac{\Lambda(t)}{t-\tau^*} - m\right) = \sqrt{t-\tau^*}\left(\frac{\Lambda(t-\tau^*)}{t-\tau^*} - m\right) + \frac{\lambda(\xi(t))\tau^*}{\sqrt{t-\tau^*}}.$$

Because $\lim_{t\to\infty}\sqrt{t-\tau^*}(\Lambda(t-\tau^*)/(t-\tau^*) - m) = 0$ is assumed and λ is bounded, condition (8.4) holds. Hence, we have proven condition (8.2), but due to $N_t - N_{\tau^*}$ and $L_{\Lambda(t)} - L_{\Lambda(\tau^*)}$ have the same distribution, for each $t > 0$, we have

$$\sqrt{t-\tau^*}\left(\frac{N_t - N_{\tau^*}}{t-\tau^*} - m\right) \xrightarrow{\mathcal{D}} \mathrm{N}(0, m), \quad \text{as} \quad t \to \infty,$$

which completes the proof. □

Proof of Theorem 5.1 From (i) in Theorem 4.2, for each $h > 0$, $\widehat{G}_\tau(h) \underset{n\to\infty}{\longrightarrow} G(h)$, $\mathbb{P}$-a.s. Hence, it follows from the Pólya Lemma (Roussas, 1997) that, as $\tau \to \infty$, $\{\widehat{G}_\tau; \tau > 0\}$ converges, $\mathbb{P}$-a.s., uniformly to the CDF G, which concludes the proof. □

Proof of Theorem 5.2 From the intermediate value theorem, there exists ξ_τ between $\widehat{m}_\tau$ and m such that $\widehat{G}_\tau(h) - G(h) = h\exp(-\xi_\tau h)(m - \widehat{m}_\tau)$. Then,

$$\sup_{h\geq 0}\left|\sqrt{\tau}(\widehat{G}_\tau(h) - G(h))\right| = \frac{\exp(-1)}{\xi_\tau}\left|\sqrt{\tau}(\widehat{m}_\tau - m)\right|.$$

However, from (i) and (ii) of Theorem 4.2, $\lim_{\tau\to\infty}\xi_\tau = m$, $\mathbb{P}$-a.s., and $\sqrt{\tau}(\widehat{m}_\tau - m) \xrightarrow{\mathcal{D}} \mathrm{N}(0, m)$, respectively. Therefore, the proof follows from the Slutsky theorem. □

References

Adelfio G, Chiodi M (2015) Alternated estimation in semi-parametric space-time branching-type point processes with application to seismic catalogs. Stoch Environ Res Risk Assess 29:443–450

Arkin B, Leemis L (2000) Nonparametric estimation of the cumulative intensity function for a non-homogeneous Poisson process from overlapping realizations. Manag Sci 46:989–998

Balakrishnan N, Stehlík M (2015) Likelihood testing with censored and missing duration data. J Stat Theory Pract 9:2–22

Breiman L (1968) Probability. Addison-Wesley, Menlo Park

Brémaud P (1981) Point processes and queues. Martingale dynamics. Springer, New York

Cornell C (1968) Engineering seismic risk analysis. Bull Seismol Soc Am 58:1583–1606

Dacunha-Castelle D, Duflo M (1986) Probability and statistics, vol 2. Springer, New York

Dargahi-Noubary G (1986) A method for predicting future large erthquakes using extreme order statistics. Phys Earth Planet Inter 42:241–245

Dieterich J (1988) Probability of earthquake recurrence with non-uniform stress rates and time-dependent failure. Pure Appl Geophys 126:589–617

Fierro R (2015) Functional limit theorems for the multivariate Hawkes process with different exciting functions. Latin Am J Probab Math Stat 12:477–489

Fierro R, Leiva V, Møller J (2015) The Hawkes process with different exciting functions and its asymptotic behavior. J Appl Probab 52:37–54

Fierro R, Leiva V, Ruggeri F, Sanhueza A (2013) On the Birnbaum-Sauders distribution arising from a non-homogeneous Poisson process. Stat Probab Lett 83:1233–1239

Fukutan Y, Suppasri A, Imamura F (2015) Stochastic analysis and uncertainty assessment of tsunami wave height using a random source parameter model that targets a tohoku-type earthquake fault. Stoch Environ Res Risk Assess 29:1763–1779

Gutenberg R, Richter C (1944) Frequency of earthquakes in California. Bull Seismol Soc Am 34:185–188

Henderson S (2003) Estimation for non-homogeneous Poisson processes from aggregated data. Oper Res Lett 31:375–382

Hristopulos D, Mouslopoulou V (2013) Strength statistics and the distribution of earthquake inter-event times. Physica A 392:485–496

Kagan Y (1997) Are eathquakes predictable? Geophys J Int 131:505–525

Kamat R (2015) Planning and managing earthquake and flood prone towns. Stoch Environ Res Risk Assess 29:527–545

Karr A (1991) Point processes and their statistical inference. Marcel Dekker, New York

Kim D, Kim B, Lee S, Cho Y (2014) Best-fit distribution and log-normality for tsunami heights along coastal lines. Stoch Environ Res Risk Assess 28:881–893

Knopoff L, Kagan Y (1977) Analysis of the theory of extremes as applied to earthquake problems. J Geophys Res 82:5647–5657

Leemis L (1991) Nonparametric estimation of the cumulative intensity function for a non-homogeneous Poisson process. Manag Sci 37:886–900

Leemis L (2004) Nonparametric estimation and variate generation for a non-homogeneous Poisson process from event count data. IIE Trans 36:1155–1160

Lewis P, Shedler G (1979) Simulation of non-homogeneous Poisson process by thinning. Nav Res Logist 26:403–413

Nicolis O, Chiodi M, Adelfio G (2015) Windowed ETAS models with application to the Chilean seismic catalogs. Spat Stat 14:151–165

Nishenko S, Buland R (1987) A generic recurrence interval distribution for earthquake forecasting. Bull Seismol Soc Am 77:1382–1399

Ogata Y (1988) Statistical models for earthquake occurrences and residual analysis for point processes. J Am Stat Assoc 83:9–27

Omori F (1895) On the after-schoks of earthquakes. J Coll Sci Imp Univ 7:111–200

Panthi A, Shander D, Singh H, Kumar A, Paudyal H (2011) Time-predictable model applicability for earthquake occurrence in northeast India and vicinity. Nat Hazards Earth Syst Sci 39:993–1002

Reid H (1910) The mechanics of the earthquake, the California earthquake of April 18-1906. Technical report 2. Carnegie Institution of Washington, Washington

Rikitake T (1976) Recurrence of great earthquakes at subduction zones. Tectonophysics 35:335–362

Roussas G (1997) A course in mathematical statistics. Academic Press, Burlington

Rubinstein J, Ellsworth W, Beeler N, Kilgore B, Lockner D, Savage H (2012) Fixed recurrence and slip models better predict earthquake behavior than the time- and slip-predictable models 2: Laboratory earthquakes. J Geophys Res 117:B02307

Shimazaki K, Nakata T (1980) Time-predictable recurrence model for large earthquakes. Geophys Res Lett 7:279–282

Sturges H (1926) The choice of a class interval. J Am Stat Assoc 21:65–66

Tucker HG (1967) A graduate course in probability. Academic Press, New York

Wesnousky S, Scholz C, Shimazaki K, Matsuda T (1984) Integration of geological and seismological data for mthe analysis of seismic hazard: a case of study in Japan. Bull Seismol Soc Am 74:687–708

Yakovlev G, Turcotte D, Rundle J, Rundle P (2007) Simulation-based distributions of earthquake recurrence times on the San Andreas fault system. Bull Seismol Soc Am 96:1995–2007